Midterm I

Name: ______________________

Closed book. One sheet of notes is allowed.

There are 8 pages of this exam including this page.

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<td>Total</td>
<td>100</td>
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### Physical Constants

<table>
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<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Electronic charge</td>
<td>$q$</td>
<td>$1.602 \times 10^{-19} \text{ C}$</td>
</tr>
<tr>
<td>Permittivity of vacuum</td>
<td>$\varepsilon_0$</td>
<td>$8.845 \times 10^{-14} \text{ F cm}^{-1}$</td>
</tr>
<tr>
<td>Relative permittivity of silicon</td>
<td>$\varepsilon_s / \varepsilon_0$</td>
<td>11.8</td>
</tr>
<tr>
<td>Relative permittivity of SiO$_2$</td>
<td>$\varepsilon_{ox} / \varepsilon_0$</td>
<td>3.9</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k$</td>
<td>$8.617 \times 10^{-5} \text{ eV K}^{-1}$ or $1.38 \times 10^{-23} \text{ J K}^{-1}$</td>
</tr>
<tr>
<td>Thermal voltage at $T = 300$K</td>
<td>$kT/q$</td>
<td>0.026V</td>
</tr>
<tr>
<td>Effective density of states (Si)</td>
<td>$N_c$</td>
<td>$2.8 \times 10^{19} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Effective density of states (Si)</td>
<td>$N_v$</td>
<td>$1.04 \times 10^{19} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Silicon Band Gap</td>
<td>$E_g$</td>
<td>1.12eV</td>
</tr>
<tr>
<td>Intrinsic carrier concentration of Si at 300K</td>
<td>$n_i$</td>
<td>$1.5 \times 10^{10} \text{ cm}^{-2}$</td>
</tr>
<tr>
<td>GaAs Band Gap</td>
<td>$E_g$</td>
<td>1.42eV</td>
</tr>
</tbody>
</table>
Problem 1: Compare Silicon with GaAs

a) Without doping, which material has a larger intrinsic conductivity at room temperature

\[ \text{Conductivity } \sigma = q.n_i.(\mu_n + \mu_p) \]

⇒ For GaAs, \( E_g \) is lower => \( n_i \) is orders of magnitude lower. (2 Pts)

(mobility for GaAs is only ~ 5 times higher)

So GaAs has lower conductivity. (1 Pts)

b) Based on band gap, estimate \( n_i^2 \) order of magnitude for GaAs.

\[ n_i^2 = N_c.N_v \cdot \exp\left(-\frac{E_g}{kT}\right) \Rightarrow \frac{n_i^2(\text{GaAs})}{n_i^2(\text{Si})} = \exp\left(\frac{E_{g,\text{Si}} - E_{g,\text{GaAs}}}{kT}\right) = 10^{-5} \] (2 Pts)

\[ \Rightarrow n_i^2(\text{GaAs}) = 10^{15} \text{ cm}^{-3} \] (2 Pts)

Assumed \( N_c \) and \( N_v \) are the same for both the materials. \( 10^{14} \text{ cm}^{-3} \) is accepted too.

c) GaAs has about the same lattice constant as Si. What is the ratio of mass densities (g/cm\(^3\)) for GaAs to that of Si. (Atomic Weights of \{(Si 28.09), (Ga 69.72), (As 74.92)\})

In a unit lattice cell ...there are 8 atoms of Si. And 4 of Ga and 4 of As in case of GaAs. Both of them are given to have same lattice contants... => Volume is same

\[ \text{Ratio of densities, } \frac{\rho_{\text{GaAs}}}{\rho_{\text{Si}}} = \frac{4 \times M_{\text{Ga}} + 4 \times M_{\text{As}}}{8 \times M_{\text{Si}}} = 0.5 \times \frac{69.72 + 74.92}{28.09} = 2.57 \] (3 Pts)

d) Assume that effective mass \( (m_n^* ) \) is 0.07\( m_0 \) for GaAs and 0.26\( m_0 \) for Si, re-estimate the \( n_i^2 \) value for GaAs.

\[ N_c \text{ would change as } N_c \propto (m_n^*)^\frac{3}{2} \]

\[ n_i^2 = N_c.N_v \cdot \exp\left(-\frac{E_g}{kT}\right) \Rightarrow \frac{n_i^2(\text{GaAs})}{n_i^2(\text{Si})} = \left(\frac{m_n^*_{\text{GaAs}}}{m_n^*_{\text{Si}}}\right)^\frac{3}{2} \exp\left(\frac{E_{g,\text{Si}} - E_{g,\text{GaAs}}}{kT}\right) \] (2 Pts)

\[ \Rightarrow n_i^2(\text{GaAs}) = 10^{14} \text{ cm}^{-3} \] (2 Pts)

\( 10^{13} \text{ cm}^{-3} \) is also accepted.

e) Assume identical mean free time, using the effective mass in d), given the mobility of electrons in Silicon is 1200cm\(^2\)/(V·s), what is the electron mobility in GaAs?
\[ \mu_n = \frac{q\tau_n}{m_n^*} \Rightarrow \mu_n(GaAs) = \mu_n(Si) \times \frac{m_n^{*}_{Si}}{m_n^{*}_{GaAs}} \quad (3 \text{ Pts}) \]

\[ \mu_n = 4457 \text{cm}^2/(V \cdot s) \quad (1 \text{ Pts}) \]

f) What's the ratio of electron diffusion constants for GaAs and Si?

Einstein Relation, \[ \frac{D}{\mu} = \frac{kT}{q} \Rightarrow D \text{ is directly proportional to } \mu. \]

As \[ \mu = \frac{q\tau}{m}, \quad \frac{D_{n,GaAs}}{D_{n, Si}} = \frac{\mu_{n,GaAs}}{\mu_{n, Si}} = \frac{m_{n, Si}^*}{m_{n, GaAs}^*} \quad (3 \text{ Pts}) \]

\[ \frac{D_{n,GaAs}}{D_{n, Si}} = 3.714 \quad (1 \text{ Pts}) \]

g) What is the ratio of the average hole kinetic energy of GaAs and Si at room temperature?

Average Hole Kinetic Energy = \[ 3kT/2 \text{ (Doesn't depend on material)} \quad (3 \text{ Pts}) \]

⇒ Ratio = 1 \quad (1 \text{ Pts})

h) Given the E-k diagram below

In general do you expect GaAs to have longer or shorter recombination lifetime, assuming both the materials have very few trap states in the band gap. Explain your conclusion concisely.

**GaAs will have a shorter recombination lifetime.** \( (2 \text{ Pts}) \)

GaAs has a direct bandgap and Si has an indirect bandgap. Recombination in Si has to happen through trap states, while in GaAs direct band to band recombination is possible.

So given we have few trap states in both the materials, the recombination lifetime is perhaps shorter in GaAs due to direct band to band jump. \( (2 \text{ Pts}) \)
Problem 2 High Power Rectifier P⁺-I-N⁺

Part A Fabrication Process

Given below is a menu consisting of various fabrication processes you are familiar with. Using only these given processes, complete the sequence of steps to fabricate the structure given below. (Hint: All given processes may or may not be required, and the same process maybe used multiple times.)

Processes Available:

<table>
<thead>
<tr>
<th>Thermal Oxidation</th>
<th>Lithography</th>
<th>Epitaxy</th>
<th>Ion Implantation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anisotropic Etching</td>
<td>Isotropic Etching</td>
<td>Chemical Vapor Deposition</td>
<td>Nano-Imprint</td>
</tr>
<tr>
<td>Sputtering</td>
<td>Chemical Mechanical Polishing</td>
<td>Thermal Annealing</td>
<td></td>
</tr>
</tbody>
</table>

Process Flow:

1. Epitaxy *(Grow Intrinsic Silicon above N⁺ Silicon Substrate)*
2. Thermal Oxidation *(Form SiO₂ thin film)*
3. Lithography (Coat photoresist, pattern contact area and develop photoresist)

4. Anisotropic Etching (Etch SiO₂ which is above P+ Si region)

5. Ion Implantation (Form P+ doping region)

6. Thermal Annealing (Repair damage and diffusion to achieve right junction depth)

7. Chemical Vapor Deposition or Sputter (Form Metal Contact)

8. Lithography (Pattern trench area of metal)

9. Anisotropic Etching (Form the trench of metal)

(2 Pts each step). You may split the formation of metal trenches in two separate steps.

Part B (doable without solving Part A)

The structure in Part A is a high voltage and high current rectifier with a 500V breakdown voltage. Assume that the critical electric field is 2x10⁵ V/cm.

a) Draw the electric field profile through the P⁺-I-N⁺ junction under reverse bias condition.

![Electric Field Profile](image)

_E-Field has to be constant in intrinsic region (3 points)...refer HW Assignment and depletion regions in P⁺ and N⁺ regions are negligible (2 points). (A threshold for the steepness of edges was set and grading was based on that => steep edges were accepted.... )_

b) What is the minimum thickness of intrinsic region to achieve the breakdown voltage?

_From the plot above all the E-Field is in the intrinsic region => All the voltage drops in this region._
Thickness, $t_{\text{intrinsic}} = \text{Breakdown Voltage} / \text{Critical Field}$ (2 points)

$$= 500 \, \text{V} / (2 \times 10^5 \, \text{V/cm}) = 2.5 \times 10^{-3} \, \text{cm}$$ (2 points)

c) What is the advantage of doping substrate N$^+$ rather than lightly doped N-type substrate?

Resistance (2 points) … More doping => lower resistance of the Quasi-Neutral region. Remember the lower part of the substrate is the second contact to this diode. This resistance tends to lower your current for a given voltage. (2 points for reason)

Capacitance increases (as $W_{\text{dep}}$ decreases), so this actually tends to make things slower. So this is not accepted as an answer.

d) What is the possible advantage of starting with N$^+$ substrate rather than P$^+$ substrate in the fabrication process in Part A?

Lower Resistance again (2 points) … mobility of holes < mobility of electrons. So the P$^+$ substrate would have a lower conductivity. (2 points for correct reason)

Question was unclear…. Everyone was awarded full marks ☺☺ ☺☺

e) Calculate the capacitance of the diode in F/cm$^2$.

Capacitance, $C = \varepsilon_0 \varepsilon_{\text{Si}} / W_{\text{dep}}$ (2 points)

But $W_{\text{dep}} = t_{\text{intrinsic}}$ (2 points)

$\Rightarrow C = 8.845 \times 10^{-14} \, \text{F} \cdot \text{cm}^{-1} \times 11.8 / 2.5 \times 10^{-3} \, \text{cm} = 4.174 \times 10^{-10} \, \text{F} \cdot \text{cm}^{-2}$ (1 point)
Problem 3 Forward-biased diode

Consider a one-sided P^+N diode as shown in the figure (uniform doping)

![Diode Diagram](image)

a. Draw the electron energy band diagram throughout the diode at $V = 0\text{V}$. Include the Fermi Level(s) in the figure.

![Energy Band Diagram](image)

*Figure (crude)... (2 points)*

*Labels important .... $E_C$, $E_V$ and $E_F$.* (2 points)

*Also relative position of $E_F$ to $E_C$ and $E_V$ on either side important (1 point)*

b. How would the band diagram look for a forward bias $V_F$ applied across the diode. Include the quasi-Fermi level(s) through the depletion region too.

![Depletion Region Diagram](image)

*Figure (crude)... (2 points)*

*Labels important .... $E_C$, $E_V$, $E_{F_n}$ and $E_{F_p}$ (2 points)*

$qV_F$ also needs to be indicated. (1 point)
c. Sketch the excess minority carrier content versus ‘x’ for two forward bias conditions, one at some value $V_F$ and another at $V_F - 60$ mV. Show the critical ratio(s).
Figure (crude) … (1 point)

Need to show \( n_p' << p_n' \) (2 points)

Labels and Ratio \( p_n' \) and \( p_n'/10 \) … (2 points)

d. Label the stored charge at \( V_F \) and \( V_F - 60 \) mV in the above question. Calculate the ratio of the stored charges at the two biases.

Label … (2 points)

Ratio … (3 points) … Need to show steps … Else (1.5 points)

For long base diode … \( Q = I \cdot \tau_p \)

\[
\frac{Q(V_F)}{Q(V_F - 60\text{mV})} = \frac{\tau_p \cdot I_0 \cdot e^{\frac{-qV_F}{kT}}}{\tau_p \cdot I_0 \cdot e^{\frac{-q(V_F-60\text{mV})}{kT}}} = e^{\frac{q60\text{mV}}{kT}} = 10, 
\]

e. Draw the minority, majority and total current densities versus \( x' \). Label the components clearly.

Label … \( J_{\text{total}}, J_{\text{maj}} \) and \( J_{\text{min}} \) atleast for lowly doped side; (2 points)
\( J_{tot} \) and \( J_{maj} \) for highly doped side. (2 points)

For indicating \( J_{min} \) on highly doped side as small / not present and \( J_{min} \) at edge of depletion for lightly doped side to be as close to \( J_{total} \). (1 point)

f. For a forward bias of 0.7 V, find the ratio of stored charge (up to the order of magnitude) at \( T = 300 \) K and \( T = 500 \) K.

Not particular about the answer but dependencies.

Temperature Dependence in \( D_p \) (2 points), \( n_i^2 \) \( (T^3 \times \exp(-E_g/kT)) \) (1 point + 1 point) and \( \exp(qV/kT) \) (1 point)

But if assumptions were made and mentioned for \( D_p \) and \( n_i^2 \) \( (T^3) \) parts… it is accepted.

\[
Q = I \times \tau \\
I = \frac{qV}{AT} - 1; I_0 = q A \frac{n_i^2}{N_D} \sqrt{\frac{D_p}{\tau_p}}; D_p = \mu_p \cdot \frac{kT}{q}
\]

\[
n_i^2 = N_e \cdot N_v \cdot \frac{e^{-E_g}}{kT}
\]

\[
\frac{Q(300K)}{Q(500K)} = \frac{I_0(300K)(e^{\frac{q \cdot 0.7}{k \cdot 300K}} - 1)}{I_0(500K)(e^{\frac{q \cdot 0.7}{k \cdot 500K}} - 1)} = \frac{n_i^2(300K)}{n_i^2(500K)} \frac{300}{500} \frac{e^{\frac{q \cdot 0.7}{k \cdot 300K}} - 1}{e^{\frac{q \cdot 0.7}{k \cdot 500K}} - 1}
\]

\[
= \left( \frac{300}{500} \right)^3 \exp \left( -\frac{q}{k} \left( \frac{1}{300} - \frac{1}{500} \right) \right) \times \sqrt{\frac{300}{500}} \times \frac{e^{\frac{q \cdot 0.7}{k \cdot 300K}} - 1}{e^{\frac{q \cdot 0.7}{k \cdot 500K}} - 1} = 1.14 \times 10^{-4}
\]

Assuming \( \mu_p \) is roughly constant