Problem 1: Semiconductor Fundamentals [20 points]

Consider a silicon sample maintained at 300K under equilibrium conditions, doped with the following impurities:
- Phosphorus: $5 \times 10^{16} \text{ cm}^{-3} = N_d$
- Boron: $5 \times 10^{16} \text{ cm}^{-3} = N_a$

a) What are the carrier mobilities in the sample? [4 pts]

\[ N_a + N_d = 10^{17} \text{ cm}^{-3} \]

From plot on Page 1, $\mu_n = 750 \text{ cm}^2/\text{V.s}$

and $\mu_p = 300 \text{ cm}^2/\text{V.s}$

\[ \mu_n = \frac{750 \text{ cm}^2/\text{V.s}}{\mu_p = \frac{300 \text{ cm}^2/\text{V.s}}{}} \]

b) What are the electron and hole concentrations in the sample? [4 pts]

\[ N_a - N_d = 0 \quad \text{(net doping concentration is zero)} \]

\[ \Rightarrow n = p = n_i \]

\[ n = \frac{10^{10} \text{ cm}^{-3}}{p = \frac{10^{10} \text{ cm}^{-3}}{}} \]

c) What is the conductivity type of the sample? [2 pts]

The sample is intrinsic.

d) What is the resistivity of the sample? [4 pts]

\[ \rho = \frac{1}{\sqrt{\mu_n n + \mu_p p}} = \frac{1}{\sqrt{\mu_n + \mu_p} n_i} \]

\[ \frac{1.6 \times 10^{-19} (750+300)(10^{10})}{5.95 \times 10^5 \text{ S/cm}} = 6 \times 10^5 \text{ S/cm} \]

\[ \rho \approx 6 \times 10^5 \text{ S/cm} \]
Problem 1 (continued)

e) What is the mean scattering time for electrons in the sample? [3 pts]
Assume \( m_n = 0.26m_o \). Note: 1 kg-cm\(^2\)/V-s/C = 10\(^{-4}\) s

\[
\mu_n = \frac{q \tau_{mn}}{m_n}
\]

\[
\Rightarrow \tau_{mn} = \frac{\mu_n m_n}{q} = \frac{(750)(0.26 \times 9.1 \times 10^{-31})}{1.6 \times 10^{-19}} = 1.1 \times 10^{-9} \text{ kg cm}^2/\text{V-s/C}
\]

\[
\tau_{mn} = 1.1 \times 10^{-13} \text{ s}
\]

f) What is the hole diffusion constant in the sample? [3 pts]

\[
D_p = \frac{kT}{q \mu_p} = (0.026)(300) = 7.8 \text{ cm}^2/\text{s}
\]

\[
D_p = 7.8 \text{ cm}^2/\text{s}
\]
Problem 2: pn Junction: Electrostatics [25 points]

Given the following electric-field distribution inside an ideal Si pn step-junction maintained at 300K:

\[
\frac{\partial \varepsilon}{\partial x} = \frac{\varrho}{\varepsilon_{si}}
\]

\[\varrho > 0 \Rightarrow \text{ionized donors}\]
\[\varrho < 0 \Rightarrow \text{ionized acceptors}\]

\[
\text{slope} = \frac{1.6 \times 10^5}{0.1 \times 10^{-4}} = 1.6 \times 10^{10} \text{V/cm}^2
\]

\[
\frac{qN_d}{\varepsilon_{si}}
\]

a) Sketch the doping profile of this pn junction. [6 pts]

\[
N_d - N_a \text{ (cm}^{-3}\text{)}
\]

\[
N_a = N_d = \frac{\varepsilon_{si} \times \text{slope}}{q} = \frac{(10^{-12})(1.6 \times 10^{10})}{1.6 \times 10^{-19}} = 10^{17} \text{ cm}^{-3}
\]

b) What is the built-in potential of this junction? [4 pts]

\[
\Phi_{bi} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}
\]

\[
= \frac{kT}{q} \ln \frac{10^{3p}}{10^{20}} = \frac{kT}{q} \ln 10^{14} = 14 \left[ \frac{kT}{q} \ln 10 \right]
\]

\[
= 14 (0.060) = 0.84 \text{ V}
\]

\[
\Phi_{bi} = 0.84 \text{ V}
\]
c) What is the applied bias $V_a$ for the given electric-field distribution? [3 pts]

$$\text{Area under the } E(x) \text{ curve} = \Phi_{bi} - V_a$$

$$\text{Area} = \frac{1}{2} (0.2 \times 10^{-4}) (1.6 \times 10^5) = -1.6 \text{ V}$$

$$1.6 = \Phi_{bi} - V_a$$

$$V_a = \Phi_{bi} - 1.6 = 0.84 - 1.6 = -0.76 \text{ V}$$

$$v_a = -0.76 \text{ V}$$

d) Sketch the energy-band diagram. Show the positions of the quasi-Fermi levels relative to $E_i$ and to each other on both sides of the junction. [5 pts]

$$E_{f_n} - E_i = kT \ln \frac{N_d}{n_i}$$

$$E_{f_p} - E_f = -qV_a = 0.76 \text{ ev}$$

e) What is the junction capacitance at this bias? [3 pts]

Under reverse bias, diffusion capacitance is negligible; therefore, junction capacitance is comprised of only the depletion capacitance.

$$C_j = C_{dep} = \frac{E_{si}}{W_{dep}} = \frac{10^{-12}}{0.2 \times 10^{-4}} = 5 \times 10^{-8} \text{ F/cm}^2$$

$$C_j = 50 \text{ nF/cm}^2$$

f) Calculate the reverse-bias breakdown voltage, assuming that the critical electric field $E_{crit}$ is $5 \times 10^5 \text{ V/cm}$. [4 pts]

$$E_p = \frac{2(\Phi_{bi} - V_a)}{W_{dep}} = \sqrt{\frac{2q(\Phi_{bi} - V_a)/N_d}{E_{si}}/E_{si}}$$

When $V_a = -V_B$, $E_p = E_{crit}$

$$E_{crit} = \sqrt{\frac{2q(\Phi_{bi} + V_B)/N}{E_{si}}/E_{si}}$$

$$V_B = \frac{E_{si}}{qN} E_{crit}^2 - \Phi_{bi}$$

$$V_B = \frac{E_{si}}{qN} \left( \frac{10^{-12}}{(1.6 \times 10^{-19})(10^{17})(5 \times 10^5)^2 - 0.84} \right) = 14.8 \text{ V}$$
a) The minority-carrier concentration profiles on the n-side of two ideal Si $p^+n$ step-junction diodes maintained at 300K are pictured below:

![Diode Concentration Profiles]

Check the appropriate boxes in the table below, and provide brief justifications for your answers. [20 pts]

<table>
<thead>
<tr>
<th>Diode Parameter</th>
<th>parameter is</th>
<th>Brief Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diode current density</td>
<td>$\sqrt{\frac{d}{dx} p_N(x)}$ is larger for Diode B</td>
<td>$\frac{d p_N}{dx} \bigg</td>
</tr>
<tr>
<td>Doping $N_d$ (n-side)</td>
<td>$\sqrt{\frac{n_i^2}{N_d}}$ is the same for both diodes</td>
<td>equilibrium value of $p_N(x)$ is the same for both diodes</td>
</tr>
<tr>
<td>Hole lifetime $\tau_p$ (n-side)</td>
<td>$\sqrt{\frac{\tau_p}{L_A}}$ is longer for Diode A</td>
<td>$p_N(x)$ has a longer $L_p$ for Diode A, $L_p = \sqrt{D_p \tau_p}$ same for both diodes, since $N_d$ is the same</td>
</tr>
<tr>
<td>Applied bias $V_a$</td>
<td>$\sqrt{p_{No} e^{-eV_a/kT}}$ is larger for Diode B</td>
<td>$p_N(x_N) = p_{No} e^{-eV_a/kT}$ is larger for Diode B</td>
</tr>
<tr>
<td>Storage delay time $\tau_s$</td>
<td>$\sqrt{\int_{x_n}^{x} (p_N(x) - p_{No}) dx}$ is larger for Diode A</td>
<td>$\int_{x_n}^{x} (p_N(x) - p_{No}) dx$ is larger for Diode A</td>
</tr>
</tbody>
</table>
Problem 3 (continued)
b) Consider the Si p⁺n step-junction diode below. The doping on the n-side is uniform, but the minority-carrier lifetime \( \tau_p \) is infinite in Region 1 and has finite value \( \tau_2 \) in Region 2. Assume that the depletion width \( W_{dep} \) is smaller than \( x_1 \), and that the length of Region 2 is much longer than the minority-carrier diffusion length \( L_2 \) in Region 2.

\[ W_{dep} \]

\[ + V_a \]

\[ \text{p⁺ Region 1} \]

\[ \tau_p = \infty \]

\[ \text{n Region 2} \]

\[ \tau_p = \tau_2 \]

\[ L_p = L_2 \]

\[ x \]

\[ 0 \]

\[ x_1 \]

\[ x_2 \]

\[ x' = x - W_{dep} \]

\[ 0 \]

\[ x_1 \]

\[ x_2 \]

i) Write expressions for the excess minority-carrier concentration in Region 1 and Region 2. [5 pts]

Note: You should express your answers in terms of several position-dependent parameters.

Region 1: minority-carrier diffusion equation:

\[ \frac{\partial p_n'}{\partial t} = D_p \frac{\partial^2 p_n'}{\partial x^2} - \frac{p_n'}{\tau_p} = D_p \frac{\partial^2 p_n'}{\partial x^2} = 0 \]

General solution is \( p_n'(x') = C_1 x' + C_2 \)

Region 2: minority-carrier diffusion equation:

\[ \frac{\partial p_n'}{\partial t} = D_p \frac{\partial^2 p_n'}{\partial x^2} - \frac{p_n'}{\tau_2} = 0 \]

General solution is \( p_n'(x') = C_3 e^{-x'/L_2} + C_4 e^{x'/L_2} \) where \( L_2 = \sqrt{D_p \tau_2} \)

Region 1: \( p_n'(x') = \frac{C_1 x'}{C_1} + C_2 \)

Region 2: \( p_n'(x') = \frac{C_3 e^{-x'/L_2} + C_4 e^{x'/L_2}}{C_3} \)

ii) Sketch the excess minority-carrier profile in the quasi-neutral n region under forward bias. Specify the boundary conditions at \( x' = 0, x_1 \) and \( x_2 \). [10 pts]

\[ p_n'(0) = \frac{n_i^2}{N_d} (e^{\frac{qV_a}{kT}} - 1) \]

\[ \frac{dp_n'}{dx'} \text{ must be continuous at } x' = x_1 \]

\[ p_n'(\infty) = 0 \]

A boundary conditions \( \rightarrow \) can solve for \( C_1, C_2, C_3 \) and \( C_4 \)

(These will be dependent on \( V_a \).) Page 7
**Problem 4: Metal-Semiconductor Contact [20 points]**

Consider an *ideal* Schottky diode maintained at 300K, made by depositing tungsten \( q\Psi_M = 4.5 \text{ eV} \) onto n-type Si.

a) What is the work function of Si, if \( N_d = 10^{17} \text{ cm}^{-3} \)? [5 pts]

\[
E_F - E_i = kT \ln \frac{N_d}{n_i} = kT \ln \frac{10^{17}}{10^{10}} = 7(kT \ln 10) = 7(0.056) = 0.42 \text{ eV}
\]

\[
E_C - E_F = \frac{1}{2} E_g - (E_F - E_i) = 0.56 - 0.42 = 0.14 \text{ eV}
\]

\[
\Psi_{Si} = \chi_{Si} + \frac{E_C - E_F}{q} = 4.05 + 0.14 = 4.19 \text{ V}
\]

\[
\Psi_{Si} = 4.19 \text{ V}
\]

b) What is the Schottky barrier height? [3 pts]

\[
\phi_{bn} = \Psi_M - \chi_{Si} = 4.5 - 4.05 = 0.45 \text{ V}
\]

\[
\phi_{bn} = 0.45 \text{ V}
\]

c) What is the built-in potential? [3 pts]

\[
\phi_{bi} = \phi_{bn} - \frac{E_C - E_F}{q} = 0.45 - 0.14 = 0.31 \text{ V}
\]

\[
\phi_{bi} = 0.31 \text{ V}
\]
Problem 4 (continued)

d) Draw the equilibrium energy-band diagram of the Schottky diode. Label $q \psi_M$, $q \psi_{Si}$, $q \lambda_{Si}$, $q \phi_{bn}$, and $q \phi_{bi}$ as well as $E_c$, $E_v$, $E_i$ and $E_f$ in the Si. [9 pts]

\[
W_{dep} = \sqrt{\frac{2e_{Si} \phi_{bi}}{q N_d}} = \sqrt{\frac{2(10^{-12})(0.31)}{(1.6 \times 10^{-19})(10^{17})}} = 6.2 \times 10^{-6} \text{ cm} = 620 \text{ Å}
\]