(1) Consider the following mechanical system where \( m_1 \) and \( m_2 \) are the masses of the blocks, \( k_1 \) and \( k_2 \) are spring constants, \( b_2 \) is the damper constant, and \( F \) is the applied force. Assume the surface is frictionless. Input to the system is the force \( F \) and the output of the system is the speed of \( m_1 \). Derive a state-variable model for this system (12%).

(2) A nonlinear system is modeled by the following differential equation where \( u \) is the input variable. The output variable of the system is \( y = (q+1)u \)

\[
\ddot{q} + (\dot{q} + q)q - 1 = u
\]

(2a) Write a nonlinear state equation for this system. (6%)
(2b) List all the equilibrium state(s) of the system with \( u = 0 \). (6%)
(2c) Linearize the system at all equilibrium states found in (2.b). The linearized equation should be in the linear state variable form (i.e., the matrix form). (8%)

(3) Consider the following system where \( u \) is the input, \( y \) is the output and \( d \) is the disturbance.

(3a) Find the transfer function from \( U(s) \) to \( Y(s) \). (8%)
(3b) Find the value of \( K \) such that the damping ratio of the complex poles of the closed-loop system is 0.5. (6%)
(4) Consider the following feedback system.

(a) What is the relationship, if any, between the poles of the transfer function from U(s) to Y(s) and the poles and zeros of \( G_f(s), G_d(s), \text{and } G_l(s) \) ? You must explain your answer. (4%)

(b) What is the relationship, if any, between the zero of the transfer function from U(s) to Y(s) and the poles and zeros of \( G_f(s), G_d(s), \text{and } G_l(s) \) ? You must explain your answer. (4%)

(c) How are the zeros of the transfer function from U(s) to Y(s) affected by the value of K as it varies from 0 to \( \infty \)? You must explain your answer. (4%)

\[
\begin{array}{c}
\text{U(s)} \\
\downarrow \\
\text{K} \\
\downarrow \\
G_f(s) \\
\downarrow \\
G_d(s) \\
\downarrow \\
Y(s)
\end{array}
\]

\( G_l(s) \)

(5) Consider the following unity feedback system.

(a) Sketch the root locus. Specifically, you must show: asymptotes and break away point (10%)

(b) Find the maximum value of K that gives the closed loop system all \textit{real} poles. (6%)

\[
\begin{array}{c}
\text{U(s)} \\
\downarrow \\
\text{K} \\
\downarrow \\
\frac{1}{s(s+6)(s+9)} \\
\downarrow \\
\text{Y(s)}
\end{array}
\]

(6) Consider the following unity feedback system.

(a) Assume \( C(s) = K \) (a constant), sketch the Nyquist plot (does not need to be precise). You must show the direction of the plot and the encirclement. (6%)

(b) Assume \( C(s) = K \), prove that the system is unstable for all positive K. (You may use any method you learned from this class). (6%)

(c) Design a compensator \( C(s) \) that stabilizes the system.

Hint: \( C(s) \) should have 1 pole, 1 zero, and a gain term K. Use the root locus concept to determine the location of pole and zero. You can then select a value for K. Your design should not involve pole-zero cancellation. You must prove that your design results in a stable closed loop system. (12%)

\[
\begin{array}{c}
\text{U(s)} \\
\downarrow \\
\text{K} \\
\downarrow \\
\text{C(s)} \\
\downarrow \\
\frac{s-2}{s} \\
\downarrow \\
\text{Y(s)}
\end{array}
\]