1. The dynamic of a nonlinear system is modeled by the following equation where \( u \) is the input variable and \( y \) is the output variable.

1.a) For \( u=0 \), there are more than one equilibrium state. One of the equilibrium states is the origin (i.e., all state variables are zero). Find another equilibrium state (for \( u=0 \)). (10%)

1.b) Linearize the system about the equilibrium found in 1.a) and express the linearized equation in the standard linear state equation form. (10%)

\[
\begin{align*}
\dot{p} &= p \cos(\dot{p}) + q(1+u) \\
\dot{q} &= p(p^2 - q) \\
y &= p(q + u)
\end{align*}
\]

2. Consider the following unity feedback system.

2.a) Give a necessary and sufficient condition on \( C(s) \) and/or \( G(s) \) such that the system is of type \( k \) (an integer) with respect to the input \( r(s) \). (10%)

2.a) Consider the following statement. If you agree with this statement, prove it. If not, explain why the statement is not true. (10%)

If this system is of type \( k \) (an integer) with respect to the input \( r \), it must be also of type \( k \) with respect to the disturbance \( d(s) \).

3. Consider the following closed loop system with a PID controller.

3.a) Assume \( K_i = 0 \), find \( K_p \) and \( K_d \) such that

- (i) the output is 0.05 at steady state when a unit step is applied to the disturbance input (and \( r(s)=0 \)) and
- (ii) the overshoot is less than 5\% at the output \( y(s) \) due to a unit step input from \( r(t) \) (5\% overshoot = 0.7 damping ratio) (10%)

3.b) Assume \( K_i = 1 \), \( K_d = 1 \), find the range of \( K_p \) such that the system is stable. (10%)

![PID Controller Diagram](attachment:image.png)
(4) Sketch the root locus for the following system. Determine all features of the root locus that apply to this particular system. Based on the root locus, find a positive value of $K$ (if such a value exist) such that the system is stable. (20%)

\[
\frac{s^2}{(s^2+1)(s+1)}
\]

(5) Plot the Nyquist plot of the following system. Use the concept of Nyquist criterion, find a positive value of $K$ (if such a value exist) such that the system is stable. (20%)

\[
\frac{s+2}{s^2(s-2)}
\]