Problem 1. (Multiple Choice 20%)

- Assume \( P(A) = 0.2, P(B) = 0.6, P(A \cup B) = 0.5 \). Then \( P[A|B] = ... \)  
  (Circle one): 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1.

- Assume that \( X = n \) with probability \( \alpha 3^{-n} \) for \( n \geq 1 \). Then \( E(X) = ... \)  
  (Circle one): 0.25, 0.5, 1, 1.5, 2, 2.5, 3, 4, 5, 6, depends on \( \alpha \)

- Assume that \( \Omega = [0, 1], \mathcal{F} = \{ \emptyset, [0, 0.3], (0.3, 1], [0, 1] \} \) and \( P([0, 0.3]) = 0.4 \). Which of the following functions of \( \omega \) are random variables? (Circle the correct answers): \( X(\omega) = 1\{\omega \leq 0.4\}, X(\omega) = 1, X(\omega) = 2 + 3 \times 1\{\omega \leq 0.3\}, X(\omega) = 1\{\omega \leq 0.5\}. \)

- Assume that \( X \geq 0, E(X) = 4, \) and \( \text{var}(X) = 0.3 \). Circle the statements that are certainly true:  
  \( P(X > 10) \leq 0.4, P(X > 10) \leq 0.5, P(X > 10) \leq 0.6, P(|X - 4| > 1) \leq 0.2, P(|X - 4| > 2) \leq 0.075, P(|X - 1| > 3) \leq 0.2. \)

- Assume that \( X \) is exponentially distributed with rate 2. Then (circle the true statements):  
  \( E(X) = 2, E(X) = 0.5, P[X > 2|X > 1] > P(X > 2), \)  
  \( P[X > 1|X > 2] < P(X > 1), P[X > 3|X > 1] = P(X > 2). \)

- Assume that \( X \) is uniformly distributed in \([0, 1]\. Then (circle the correct statements):  
  \( E(\cos(X)) = \sin(1), E(X^n) = 2n, E(X^n) = 1/(n + 1), \)  
  \( \text{var}(X) = 1/4, \text{var}(X) = 1/12. \)

- Let \((X, Y)\) be uniformly picked in the triangle with vertices \((0, 0), (0, 1), (1, 0)\). Circle the correct statements:  
  \( X, Y \) are independent. \( X, Y \) are positively correlated. \( X, Y \) are uncorrelated.

- Complete the following sentence: A random variable is a ....
Problem 2. (20%) A random number generator of type $n$ selects a number uniformly in the set $\{1, 2, \ldots, n\}$. You are given such a generator and you are told that its type is $n$ with probability $n2^{-n-1}$ for $n \geq 1$. Let $X$ be the value that the random number generator selects. Given $X$, what is the probability that the type of the generator is $n$, for $n \geq 1$?
Problem 3. (20%) The point \((X, Y)\) is picked uniformly in the triangle with vertices
\((0, 0), (0, 1), (1, 1)\). Calculate \(\text{cov}(X, Y)\), the covariance of \(X\) and \(Y\). Recall that
\[\text{cov}(X, Y) = E(XY) - E(X)E(Y).\]
Problem 4. (20%) Let $X$ be the random variable selected as follows: with probability 0.2, $X = 0.8$; with probability 0.3, $X = 1.4$; with probability 0.5, $X$ is picked uniformly in $[0, 2]$.
(a) Give the pdf $f(x)$ of $X$; (b) Give the cpdf $F(x)$ of $X$; (c) Calculate $E(X)$; (d) Calculate $\text{var}(X)$. 
Problem 5. (10%) Let $F(x)$ be the cpdf of a random variable $X$. Prove that $F(x)$ is right-continuous.

Problem 6. (10%) For $n \geq 1$, let $X_n(\omega) = 1\{\omega \in A_n\}$ where the events $A_n$ are such that $P(A_n) = 1/(4n^2)$. Show that $X := \sum_{n=1}^{\infty} X_n$ is finite with probability 1.