1. (15%)  
There are two coins. Coin 1 is fair. Coin 2 is such that \( P(H) = 0.6 \).  
1) You flip the two coins together repeatedly. What is the probability that coin 1 yields \( H \) before coin 2?  
2) You are given one of the two coins, with equal probabilities. You flip the coin twice and you get \( H \) both times. What is the probability that you got coin 1?
2. (20%)  
You throw a dart at a circular target with radius 1. You miss the target with probability 0.2. If you hit the target, the dart location is uniformly distributed inside the target. Let $X$ be the distance from dart to the center of the target when you hit it and $X = 2$ when you miss the target.  
1) What is the p.d.f. of $X$;  
2) Plot the c.p.d.f. of $X$;  
3) Calculate $Var(X)$, the variance of $X$.  

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3. (20%) You pick a point $\omega$ uniformly in the square $[0, 1]^2$ and you designate the coordinates of the point by $X(\omega)$ and $Y(\omega)$.

1) Calculate $E(|X - Y|)$.

2) Calculate $P[X \leq x \mid |X - Y| > 0.5]$ for $x \in [0, 1]$. 
4. (15%) Assume that humanity will either survive 10 billion years or ten million years, with equal probabilities. For simplicity, assume that the population is constant and about equal to 8 billion people, in both cases. Assume also that you are picked randomly human, among all humans who will ever live. You observe that humanity has been around for about 5 million years. What is the probability that humanity will survive ten billion years, given your observation?
5. (15%) 
A randomly picked 126 student has a 20% chance of being a genius and an 80% chance of being very smart but somehow short of genius. A genius gets a score on the first midterm that is uniformly distributed in \([70, 100]\). A very smart student gets a score that is uniformly distributed in \([0, 100]\). A genius has a probability 80% of going to graduate school and a very smart student has a probability 20% of going to graduate school. What is the probability that a randomly picked student who gets a score of 80 will go to graduate school?
6. (15%)
Assume that $P(X = n) = (1 - p)^{n-1} p, n \geq 1$ where $p \in (0, 1)$. Calculate $E(X^k)$ for $k \geq 1$. 