Department of EECS - University of California at Berkeley
EECS126 - Probability and Random Processes - Spring 2003
Midterm No. 2: 4/08/2003

Name and SID:

There are seven problems. Answer on these sheets. Show your work. Good luck.

Problem 1 (10%). Give an example of a pair of random variables (X, Y) that are uncorrelated and not independent.

Problem 2 (10%). Give an example of a pair of random variables (X, Y) that are not independent and are such that $E[X|Y] = E(X)$.

Problem 3 (10%). Is it possible for a pair of random variables (X, Y) to be such that $E[X|Y] > X$ for all Y? Explain your answer.

Problem 4 (10%). Let X, Y, Z be independent and uniformly distributed on $[-1, 1]$. Calculate $E[X + Y|X + Y + Z]$.

Problem 5 (15%). Let X, Y, Z be independent and equally likely to take the values $\{-2, -1, 0, 1, 2\}$. Calculate $L[X + 2Y|X + Y, Y + Z]$.

Problem 6 (25%). Let X, Z be independent with $P(X = 0) = 0.4, P(X = 1) = 0.6$, and $Z = N(0, 1)$. Find the MLE and the MAP of X given $Y = X + (1 + X)Z$.

Problem 7 (30%). For $x = 0, 1$, given $X = x$, Y is exponentially distributed with mean $\mu(x)$, for $x = 0, 1$ where $0 < \mu(0) < \mu(1)$.

a. Find $\hat{X} = g(Y)$ that maximizes $P[\hat{X} = 1 | X = 1]$ subject to $P[\hat{X} = 1 | X = 0] \leq 5\%$.

b. Assume that $\mu(0) = 1$. Find the minimum value of $\mu(1)$ so that $P[\hat{X} = 1 | X = 1] \geq 95\%$.