Department of EECS - University of California at Berkeley
EECS126 - Probability and Random Processes - Spring 2003
Midterm No. 1 bis: 4/16/2003

Name and SID:

There are four short questions and six slightly longer problems. Answer on these sheets. Show your work. Good luck.

Question 1 (6%). Over a large number of experiments, you watch the number of photons that hit a photodetector in one hour. You record the fraction of experiments when the number of photons is even and when it is odd. You find that the probability that the number is even is 0.6.

a. Describe the probability space $\{\Omega, \mathcal{F}, P\}$ that models that information.

b. Give an example of a function $X : \Omega \to \Re$ that is not a random variable for that probability space.

c. Give a different probability space with the same Ω such that the same function $X(\omega)$ is a random variable on that space.

Question 2 (6%). Let $\{\Omega, \mathcal{F}, P\}$ be the probability space that corresponds to rolling a balanced die. Give an example of two events A and B in that probability space that are disjoint and independent.

Question 3 (8%). Let X be a Poisson random variable with mean 1. Calculate $P(X \text{ is even})$.

Question 4 (6%). Assume that A, B, C are events in $\{\Omega, \mathcal{F}, P\}$ that are mutually independent. Prove that

$$P[A|B^c \cap C] = P(A).$$

Problem 1 (8%). You roll a balanced die 4 times. Let X be the sum of the faces of the first and second tosses, Y be the sum of the faces of the second and third tosses, and Z be the sum of the third and fourth tosses. Calculate $E(\max\{X, Y, Z\})$.

Problem 2 (6%). Let X and Y be two independent random variables that are exponentially distributed with mean 1. Calculate $E(\max\{X, 2Y\})$.

Problem 3 (10%). Let X, Y be independent and uniformly distributed in $[0, 1]$. Find $E(\min\{2X - Y, X + Y\})$.

Problem 4 (8%). You pick a point in the unit circle with the uniform distribution. Designate the Cartesian coordinates of the point by (X, Y). Find $P(X > 3Y)$.

Problem 5 (12%). You pick a point X in the unit interval $[0, 1]$ with the uniform distribution. Plot the c.d.f. of $Y = \max\{0.2, |X - 0.4|\}$. Calculate $E(Y)$ and $var(Y)$.

Problem 6 (10%). There are two coins. One coin is fair, the other is biased with $P(H) = 0.6$. You are given one of the two coins, the fair one with probability 0.7 and the biased coin with probability 0.3. What is the probability that you got the fair coin given that after tossing it you get 'H'?
