Department of EECS - University of California at Berkeley
EECS126 - Probability and Random Processes - Spring 2000
Midterm No. 2: 4/5/2000

Name and SID:

Answer the questions on these four sheets. Show your work. Good luck.

Problem 1: Let X, Y be independent standard Gaussian random variables.
Calculate $E[(X+Y)^4 | X-Y]$.

Problem 2: Let X, Y be independent random variables uniformly distributed in $[0, 1]$. Calculate $E[X|X < Y]$.

Problem 3: Let X, Y be independent random variables uniformly distributed in $[0, 1]$. Calculate $E[X|X^2 + Y^2]$ and the LLSE of X given $X^2 + Y^2$.

Problem 4: A machine produces steel balls for ball bearings. When the machine operates properly, the radii of the balls are i.i.d. and $N(100, 4)$. When the machine is defective, the radii are i.i.d. and $N(98, 4)$.

a. You measure n balls produced by the machine and you must raise an alarm if you believe that the machine is defective. However, you want to limit the probability of false alarm to 1%. Explain how you propose to do this.

b. Compute the probability of missed detection that you obtain in part (a). This probability depends on the number n of balls, so you cannot get an explicit answer. Select the value of n so that this probability of missed detection is 0.1%.

To solve this problem you need to use some of the following information: Let $Q(x) := P(N(0, 1) > x)$. Then,

$$Q(1.6) \approx 5\%; Q(2.3) \approx 1\%; Q(2.6) \approx 0.5\%; Q(3.1) \approx 0.1\%; Q(3.3) \approx 0.05\%; Q(3.7) \approx 10^{-4}.$$