Name and SID:

Answer the questions on these four sheets. Show your work. Good luck.

Problem 1: (25%) You flip a fair coin repeatedly. What is the probability that you have to flip it exactly 10 times to see two “heads”? 
Problem 2: (25%) Let \( A, B, C \) be three events. Assume that 
\[
P(A) = 0.6, \quad P(B) = 0.6, \quad P(C) = 0.7, \quad P(A \cap B) = 0.3, \quad P(A \cap C) = 0.4, \quad P(B \cap C) = 0.4, \quad P(A \cup B \cup C) = 1.
\]
Find \( P(A \cap B \cap C) \).
Problem 3: (25%) There are two coins. The first coin is fair. The second coin is such that \( P(H) = 0.6 = 1 - P(T) \). You are given one of the two coins, with equal probabilities between the two coins. You flip the coin four times and three of the four outcomes are \( H \). What is the probability that your coin is the fair one?
Problem 4: (25%) Define the random variable \( X \) as follows. You throw a dart uniformly in a circle with radius 5. The random variable \( X \) is equal to 2 minus the distance between the dart and the center of the circle if this distance is less than or equal to one. Otherwise, \( X \) is equal to 0.

a. Plot carefully the probability distribution function \( F(x) = P(X \leq x) \) for \( x \in \mathbb{R} := (-\infty, +\infty) \).

b. Give the mathematical expression for the probability density function \( f(x) \) of \( X \) for \( x \in \mathbb{R} := (-\infty, +\infty) \).