

EECS 126 -- MIDTERM # 1 Professor Ren

October 9 , 1997, Thursday 6-8 p.m.

**[20 pts.] 1. Given  $P(A) = \alpha, P(B) = \beta$** 

$$P(A \cap B) = \gamma$$

Find: **a)  $P(A^c \cap B^c)$** 

**b)  $P(A^c \cup B^c)$**

**c)  $P(A^c|B)$**

**d)  $P(A^c|B^c)$**

[15 pts.] 2. A committee of four is picked randomly from a pool of 5 men and 4 women. Find the probability that there will be more women than men on the committee.

[25 pts ] 3. Given two coins with probability of heads being  $p_1$  for coin 1, and  $p_2$  for coin 2. You randomly pick a coin and flip it.

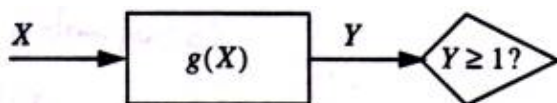
Let:  $X$  = the number of heads in  $n$  flippings of the randomly picked coin.

$Y$  = the number of flippings it takes to get the first head (flipping the randomly picked coin).

a) Find the probability mass functions of  $X$  and  $Y$ , respectively.

b) Suppose you flipped  $k$  times already and still have not got a head yet. Find the probability that you picked coin 1.

[40 pts] 4. Consider a signal detector to detect if a signal is present or not, as shown below:



where

$X$  is the received signal plus noise, and

$X = \begin{cases} S, & \text{when the signal is present (with probability } 1/2) \\ M, & \text{when the signal is not present (with probability } 1/2) \end{cases}$

$S$  is a uniform random variable in  $[-2,2]$ , and  $M$  is a Gaussian RV with distribution  $N(0,1)$ .

- a) Find the pdf of  $X$ .
- b) Let  $g(X) = |X - 1|$ . Find the pdf of  $Y = |X - 1|$ .
- c) Given that  $Y \geq 1$ , find the probability that the signal is present.