Department of EECS - University of California at Berkeley

EECS 126 - Probability and Random Processes - Fall 2008

Midterm 2: 11/18/2008

Name (Last, First):

SID:

---

1. Definition (10%)

Define "Jointly Gaussian Random Variables"

---

2. Orthogonality (10%)

Give an example of a two orthogonal random variables that are not independent.

3. Gaussian but not jointly (10%)

Give an example of two $N(0,1)$ random variables that are not jointly Gaussian.

---

4. Conditional Expectation (10%)

Is it true that $E[X|Y] = 0$ implies that X and Y are uncorrelated? Prove or provide a counterexample.

5. Conditional Expectation, again (10%)

Let X, Y, Z be i.i.d. and uniformly distributed in $[0, 1]$. Calculate $E[(X+Y)^2|Y+Z]$.

6. Flipping coins (10%)

You flip a coin n times. The probability p that a coin toss yields H is uniformly distributed in $[0, 1]$. Calculate the variance of the number of Hs in the n tosses.

7. Jointly Gaussian (15%)

Let (X, Y) be jointly Gaussian, zero mean, with $\text{var}(X) = 4$, $\text{var}(Y) = 1$ and $\text{cov}(X, Y) = 1$. Calculate $E[X^2|Y]$.

8. Jointly Gaussian, again (15%)

Assume that $(X, Y_1, Y_2)^T = N(\mathbf{m}, \Sigma)$ with

$$\mathbf{m} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 6 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}.$$

Calculate $E[X|Y_1, Y_2]$.

9. Detection and Hypothesis Testing (10%)

Given $X \in \{0, 1\}$, the random variable Y is exponentially distributed with rate $3X + 1$ (thus, with mean $(3X + 1)^{-1}$).

1) Assume $P(X = 1) = p, P(X = 0) = 1 - p$. Find the MAP estimate of X given Y.

2) Find the MLE of X given Y.

3) Solve the hypothesis testing problem of X given Y with a probability of false alarm at most 10%. That is, find $\hat{X}$ as a function of Y that maximizes $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq 0.1$.

4) For what value of p does one have the same solution for 1) and 3)?