Department of EECS - University of California at Berkeley

EECS 126 - Probability and Random Processes - Fall 2008

Midterm 1: 10/09/2008

Name:

SID:

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1. Short Questions (20%); 4% each

1.1. Define "Random Variable"

1.2. Complete the sentence: A, B, C are mutually independent if and only if

1.3. Bayes's Rule. Assume that $\{A_1, \ldots, A_n\}$ form a partition of Ω and that $p_m = P(A_m)$ and $q_m = P[B|A_m]$ for $m = 1, \ldots, n$. Derive $P[A_m|B]$ in terms of p's and q's.

1.4. Assume that X is equal to 2 with probability 0.4 and is uniformly distributed in $[0, 1]$ otherwise. Calculate $E(X)$ and $\text{var}(X)$. (Hint: Recall that $\text{var}(X) = E(X^2) - E(X)^2$.)

1.5. Two random variables X, Y are related so that $aX + Y = b$ for some real constants a and b. Given $E(X) = \mu$, $\text{var}(X) = \sigma^2$, express $E(Y)$ and $\text{var}(Y)$ in terms of μ and σ.

2. Posterior Probability (10%)

Two manufacturing plants produce similar parts. Plant 1 produces 1000 parts, 100 of which are defective. Plant 2 produce 2000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant 1?

3. Posterior Probability (10%)

A number is selected at random from 1,2,...,100. Given that the number selected is divisible by 2, what is the probability that the number is divisible by 3 or 5?

4. Expectation (15%)

There is a series of mutually independent Bernoulli experiments that individually have probability p of success and probability $(1-p)$ of failure. These experiments are conducted until the r^{th} success. Let X be the number of failures that occur until this r^{th} success. The pmf of X is:

$$p_X(k) = \binom{k+r-1}{k} p^r (1-p)^k, k \geq 0$$

a) Justify the pmf.

b) Express $E(X)$ in terms of p and r.

5. Independence (15%)

Show that if three events A, B, and C are mutually independent, then A and $B \cup C$ are independent.

6. Probability Distribution (10%)

Find the allowable range of values for constants a and b such that the following function is a valid CDF

$$F(x) = 1 - ae^{-x/b} \text{ if } x \geq 0, \text{ and 0 otherwise.}$$

For those values of a and b, compute $P(-2 < X < 10)$ where X is the associated random variable.

7. Preemptive Maintenance (20%)

Assume, for simplicity, that the lifetime of a machine is uniformly distributed in $[0, 1]$. If the machine fails, you face a cost equal to C, which includes replacing the machine with a new one. Replacing the machine before it fails with a new one costs $K < C$. You decide to replace the machine after $T < 1$ time units or when it fails, whichever comes first.

(a) Let X be the random time when you replace the machine. Calculate $E(X)$ in terms of T.

(b) Let Y be the random cost when you replace the machine (either K or C). Calculate $E(Y)$ in terms of T.

(c) The average replacement cost per unit of time is $E(Y)/E(X)$. Find the value of T that minimizes that average cost.