The exam starts at 3:40 p.m. sharp and ends at 5:00 p.m. sharp.
There are 5 problems. The maximum score is 50 points.
The exam is open book and notes.

**Problem #1 - 25 points**
For each of the following statements, indicate whether you believe that the statement is true or believe it is false, and give a brief explanation of your reasoning. A correct answer without a valid explanation gets 1 point. A correct answer with a valid explanation gets 5 points.

(a) If $X$ is a Gaussian random variable the $X$, $2X$, and $3X$ are jointly Gaussian random variables.

(b) Let $X$, $Y$, and $Z$ be random variables, which you may assume have a joint density. Let $W = Y + Z$. Then
$$E[W | X] = E[Y | X] + E[Z | X]$$

(c) Let $X$, $Y$, and $Z$ be random variables, which you may assume have a joint density. Let $W = Y + Z$. Then
$$E[X | W] = E[X | Y] + E[X | Z]$$

(d) If $X$ is Gaussian and $Y$ is uncorrelated with $X$, then $X$ and $Y$ are independent.

(e) If Gaussian random variables $X$ and $Y$ have the same mean and the same second moment then they have the same fourth moment.

**Problem #2 - 7 points**
Let $X \sim N(2,2)$. Let the conditional density of $Y$ given $X$ be given by
$$f_{Y|X}(y|x) = \frac{1}{(3/2)^{1/2} \cdot (2 \cdot \pi)^{1/2}} \cdot e^{-1/2 \cdot (2/9) \cdot (y - (1/2)(x-2) - 3)^2},$$
i.e. conditional on $X$, $Y$ is Gaussian with mean $(1/2)(X - 2) + 3$ and variance $9/2$. Find the density of $Y$.

**Problem #3 - 6 points**
I throw three darts at a disk of radius $R_0$ centered at the origin. Each dart lands on the disk and the point at which it lands is distributed according to
$$f_{R, \Theta}(r, \theta) = \begin{cases} 
(3 \cdot r^2 / (2 \cdot \pi \cdot R_0^3)) & \text{if } 0 \leq r \leq R_0 \text{ and } -\pi \leq \theta \leq \pi \\
0 & \text{elsewhere}
\end{cases}$$
Problem #4 - 7 points
X and Y are jointly Gaussian mean zero random variables. You are told that the variance of X + Y is 1, the variance of X - Y is 1, and the Cov(X + Y, X - Y) = 0. Find the joint density of X and Y.

Problem #5 - 5 points
X and Y are independent random variables, each of which is exponentially distributed with parameter 1. Find the linear MSE estimate of X + Y given X - Y.