Midterm Exam 1 Solutions

1)

$$P(A|B) = a \; P(B) = b \; P(B^c|A^c) = e$$

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)} = \frac{ab}{P(A)}$$

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c) = ab + P(A|B^c)(1-b)$$

$$P(A|B^c) = 1 - P(A^c|B^c)$$

$$P(A^c|B^c) = \frac{P(B^c|A^c)P(A^c)}{P(B^c)} = \frac{e(1-P(A))}{1-b}$$

Plugging in the previous 3 equations and algebraic simplification to get P(A) gives

$$P(A) = 1 + \frac{ab}{1 + \frac{b(a-1)}{1-e}} = \frac{ab(1-e)}{1-e+ab-b}$$

2) a)

p(active)=$\frac{1}{3} = p$

$$pdf = f_X(x) = \sum_{k=0}^{48} \binom{48}{k} p^k (1-p)^{48-k} \delta(x-k)$$

$$cdf = F_X(x) = \sum_{k=0}^{x} \binom{48}{k} p^k (1-p)^{48-k} u(x-k)$$

b)

$$P(X > 24) = \sum_{k=25}^{48} \binom{48}{k} p^k (1-p)^{48-k}$$

c) The expected fraction of dropped packets = number of dropped packets over the number of total packets produced.

$$= \frac{\sum_{k=M+1}^{48} (k-M)\binom{48}{k} p^k (1-p)^{48-k}}{\sum_{k=0}^{48} \binom{48}{k} k p^k (1-p)^{48-k}}$$

$$= \frac{\sum_{k=M+1}^{48} (k-M)\binom{48}{k} p^k (1-p)^{48-k}}{np}$$

3)

The last flip must be heads, but the other two heads can come anywhere n the previous 7 flips as long as 2 and only 2 heads show up.

$$p = 0.6 * \binom{7}{2}(0.6)^2(0.4)^5$$

4)
5 accidents/month gives 60 accidents per year

$$P(X = 0) = e^{-60}$$

5)
Acceptable outcomes for a win are a 6 on the following rolls 1,3,5,7,...
The probability of rolling a 6 is $p = \frac{5}{36}$

$$P(win) = \sum_{k=0}^{\infty} p(1-p)^{2k} = p\sum_{k=0}^{\infty}(1-p)^{2k}$$

$$= \frac{\frac{5}{36}}{1 - \left(\frac{31}{36}\right)} = 0.537$$

6)

$$P(A|B) < P(A)$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} < P(B)$$

Therefore, less likely.