EECS 123 -- MIDTERM 1

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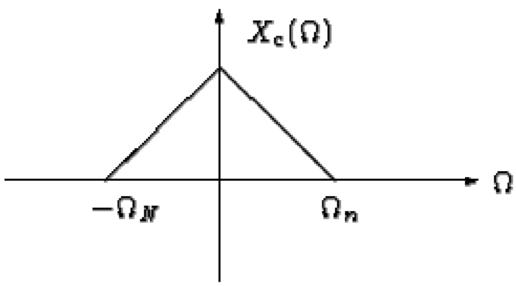
4 October 1994
8:10 - 9:30 a.m.

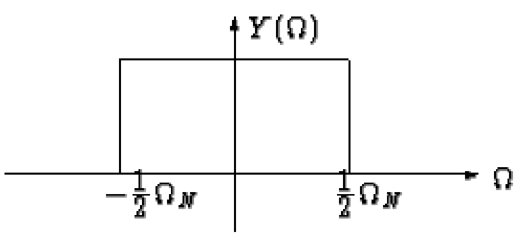
This is an open-book exam. Calculators are allowed. Please show your work *clearly* if you wish to receive partial credit. Good luck!

1.

Consider sampling of a continuous-time signal **xc** (**t**), with a Fourier transform $X_c(\Omega)$ which is real and shown below:



Discrete-time processing is done with a filter having impulse response **h**[**n**] and discrete-time Fourier transform $H(e^{j\omega})$. Reconstruction uses an ideal lowpass filter $\mathbf{h}_1$ (**t**). One wants to obtain an output signal **y**(**t**) with Fourier transform $Y(\Omega)$ which is real and given by



(a)

15 pts. Sample **x**(**t**) with sampling frequency $\Omega_s = 2 \cdot \Omega_N$, filter the resulting sequence **x**[**n**] with **h**[**n**], and reconstruct the output using an ideal lowpass $\mathbf{h}_1$(**t**) with cut-off frequency Ω_N and gain $\frac{2\pi}{\Omega_s} = T$.

i.

10 pts. Sketch and give the formula for $H(e^{j\omega})$ such that $Y(\Omega)$ is obtained.

ii.

5 pts. If $\mathbf{h}_1$(**t**) has cut-off frequency $\frac{1}{2}\Omega_N$ instead, can $H(e^{j\omega})$ be modified, and if so, how? (Hint: There will be a don't care region.)

(b)

15 pts. Now sample $\mathbf{x}(\mathbf{t})$ with sampling frequency $\Omega_s = \frac{3}{2}\Omega_n$.

i.

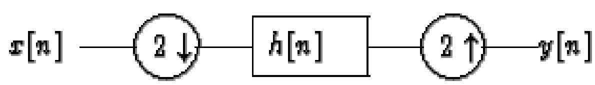
8 pts. Give the spectrum $X_s(\Omega)$ in this case.

ii.

7 pts. Indicate how to reconstruct as much of the spectrum of $X(\Omega)$ as possible from $X_s(\Omega)$, by interpolation from the samples. Give the interpolation filter $H_i(\Omega)$.

2.

Consider the following multirate system, that is, involving sampling rate changes



where $2\downarrow$ and $2\uparrow$ mean downsampling and upsampling by 2, respectively.

(a)

10 pts. Is the above system

i.

linear?

ii.

time-invariant?

iii.

causal?

(b)

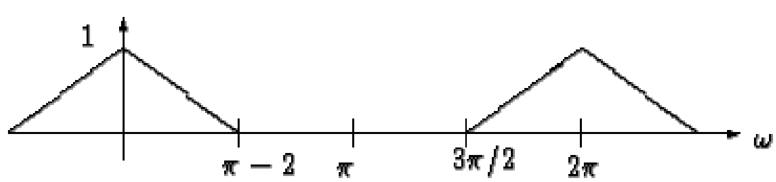
10 pts. Consider $h[n] = \delta[n]$, that is, the identity. What is $\mathbf{y}[\mathbf{n}]$as a function of $\mathbf{x}[\mathbf{n}]$? (Do this in time-domain.)

(c)

10 pts. Consider now discrete-time signals $\mathbf{x}[\mathbf{n}]$ which are half-band lowpass, namely

$$X(e^{j\omega}) = 0 \qquad \pi/2 \leq |\omega| \leq \pi$$

For example, one such signal is given by



Now, the filter $\mathbf{h}[\mathbf{n}]$ is an ideal half-band lowpass filter with

$$H(e^{j\omega}) = \begin{cases} 2 & |\omega| < \pi/2 \\ 0 & |\pi/2| \le |\omega| \le \pi \end{cases}$$

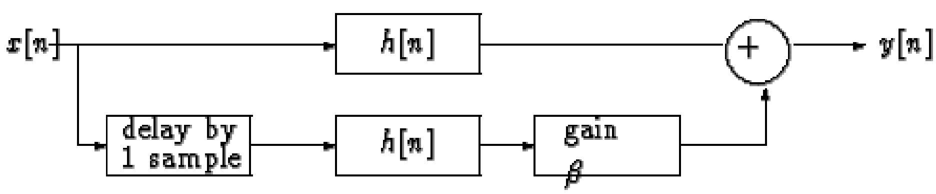
What is the output $Y(e^{j\omega})$ for the example input signal $X(e^{j\omega})$ shown above?

(d)

10 pts. The output **y**[**n**] is now lowpass filtered by an ideal half-band filter $H_{LP}(e^{j\omega}) = 1$ for $|w| < \pi/2$, 0 for $\pi/2 \le |w| \le \pi$. The output of **HLP** is called **z**[**n**]. For input signals that are half-band lowpass, show that the input-output relationship from **x**[**n**] to **z**[**n**] is that of an ideal lowpass filter. What is its cut-off frequency?

3.

Consider the LTI system:



(a)

5 pts. Find an expression for the transfer function $A(z) = \frac{Y(z)}{X(z)}$ in terms of **H**(**z**).

(b)

5 pts. When $x[n] = \delta[n]$, the output is $f[n] = \alpha^n u[n] + \beta \cdot \alpha^{n-1} u[n]$. Find **h**[**n**]and **H**(**z**) in terms of α and β.

(c)

5 pts. Plot the region of convergence of **H**(**z**).

(d)

5 pts. Find the DTFT of **h**[**n**]. For what values of α does $H(e^{j\omega})$ uniformly converge?

(e)

5 pts. For $\alpha = \frac{1}{2}$ and $\beta = -2$, plot the pole-zero diagram of $A(z) = \frac{Y(z)}{X(z)}$. Include the unit circle in your diagram.

(f)

5 pts. For $\alpha = \frac{1}{2}$ and $\beta = -2$, plot the magnitude of $A(e^{j\omega})$. (Hint: Express the numerator as a complex number times the complex conjugate of the denominator. The answer is very simple.)

1998-11-22 Converted to HTML by Danny Yoo (dyoo@uclink4.berkeley.edu)