Problem #1 (30 pts)

A band-limited signal x(t) whose spectrum is specified in Figure 1 is fed into the system described on Figure 2. Note that

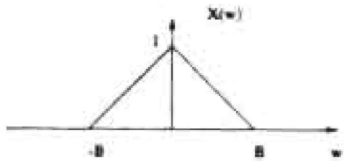

Figure 1: Problem 1: $X(w)$

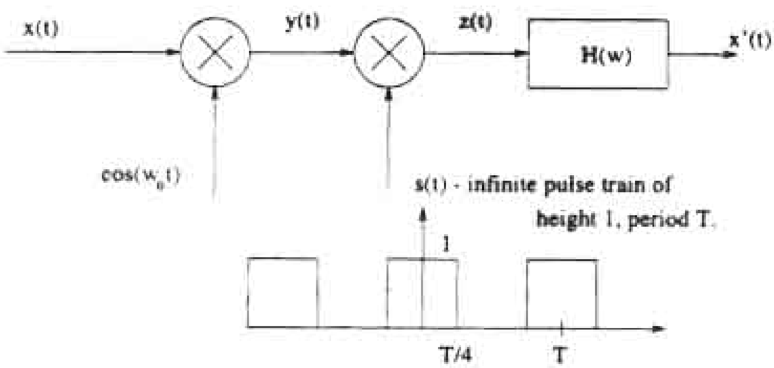

Figure 2: Problem 1: System diagram.

$$w_0 = 2B \qquad T = \frac{\pi}{B}$$
$$y(t) = x(t)cos(w_0 t)$$
$$z(t) = y(t)s(t)$$

where s(t) is infinite pulse train with period T, height 1, and 50% duty cycle.

a) (6 pts)

Find Y(*w*). Sketch Y(*w*) and indicate **all** salient features (heights of the peaks and intersections of Y(*w*) with *w*-axis).

b) (8 pts)

Find S(*w*). Sketch S(*w*) in the range -4B < *w* < 4B, and indicate **all** salient features (heights of the peaks and intersections of S(*w*) with *w*-axis).

c) (10 pts)

Sketch Z(*w*) in the range -3B < *w* < 3B, and indicate **all** salient features (heights of the peaks and intersections of Z(*w*) with *w*-axis).

d) (6 pts)

Find H(w) such that x'(t) = x(t).

Problem #2 (35 pts)

Let f(t) be a periodic function with period 2*pi, shown of Figure 3 and specified by

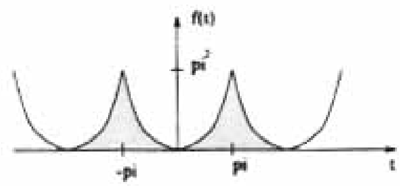

Figure 3: Problem 2: $f(t)$

$$f(t) = t^2 \qquad -\pi < t < \pi$$

The Fourier Series expansion of $f(t)$ is given by

$$f(t) = \frac{\pi^2}{3} - 4\left(\frac{cos(t)}{1^2} - \frac{cos(2t)}{2^2} + \frac{cos(3t)}{3^2} \ldots\right)$$

Now consider (**non** periodic) function $g(t)$, shown on Figure 4 and specified by

$$g(t) = \begin{cases} (\frac{t}{2\pi})^2 & -2\pi < t < 2\pi \\ 0 & otherwise \end{cases}$$

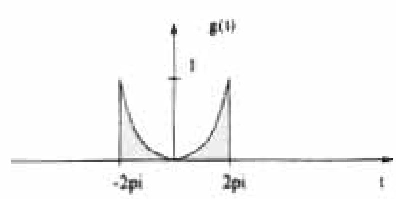

Figure 4: Problem 2: $g(t)$

a) (15 pts)

Find X(w) and Y(w) such that G(w) equals to convolution of X(w) and Y(w), i.e. G(w) = X(w) * Y(w)

b) (20 pts)

If G(w) is expressed as

$$G(\omega) = \sin(2\pi\omega)\sum_{k=0}^{\infty} a(\omega, k)$$

find function a(w,k).

Problem #3. (35 pts)

NOTE: In this problem, a black circle at position (x0, y0) signifies delta(x - x0)delta(y-y0).

a) (7 pts)

Find the 2D Fourier Transfore of function

$$f_1(x,y) = \sum_{k=-\infty}^{\infty} \delta(x-k)\delta(y)$$

also drawn in Figure 5.

Sketch $F_1(w_x, w_y)$.

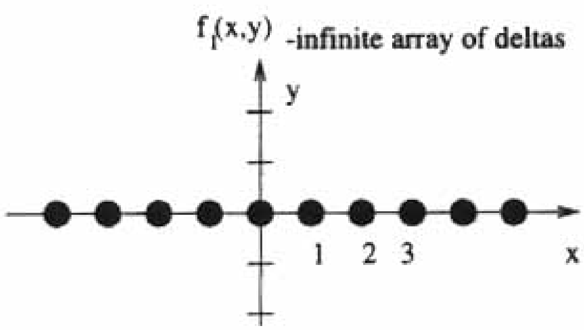

Figure 5: Problem 3: $f_1(t)$

b) (8 pts)

Find the 2D Fourier Transform of function

$$f_2(x,y) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \delta(x-2k)\delta(y-l)$$

also drawn in figure 6.

Sketch $F_2(w_x, w_y)$.

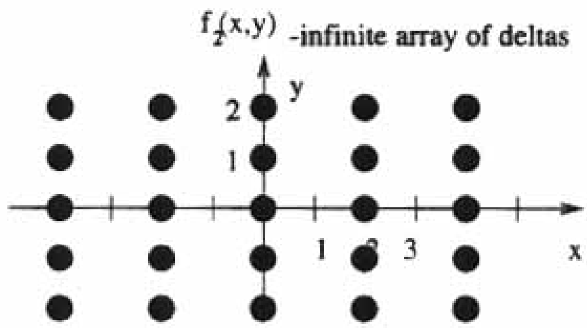

Figure 6: Problem 3: $f_2(t)$

c) (10 pts)

Find the 2D Fourier Transform of function

$$f_3(x,y) = \sum_{k+l \text{ is even}} \delta(x-k)\delta(y-l)$$

also drawn in figure 7.

Sketch $F_3(w_x, w_y)$.

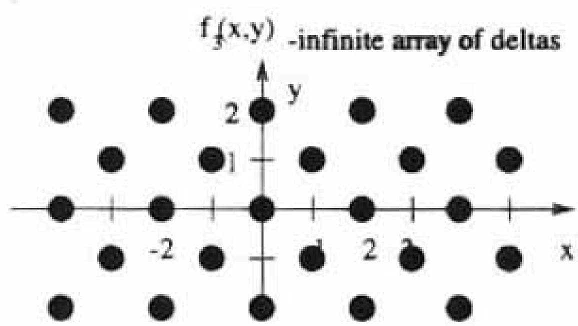

Figure 7: Problem 3: $f_3(t)$

d) (10 pts)

Find the 2D Fourier Transform of function $f4$(x,y) also drawn in figure 8. Note that $f4$(x,y) is equal to 1 in shaded areas, and to 0 otherwise.

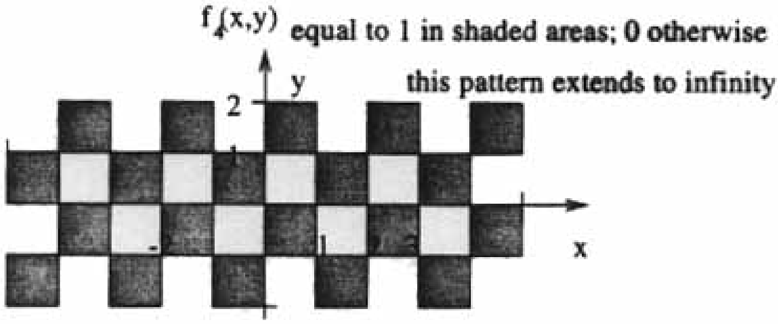

Figure 8: Problem 3: $f_4(t)$