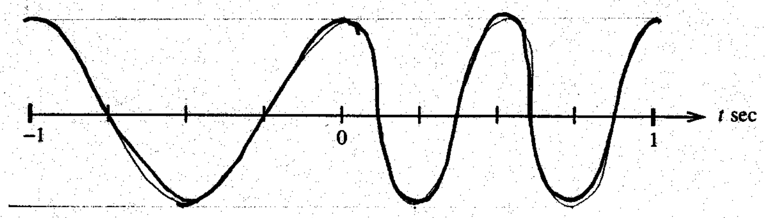
SPRING '97 FINAL

Problem 1 (10 points)

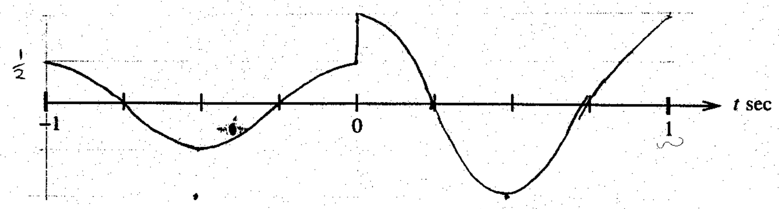
Sketch the following time functions, labelling maximum amplitude.

[4 pts.] **a)** $\cos[2\pi t + 2\pi t u(t)]$

$t<0: \cos(2\pi t)$
$t>0: \cos(4\pi t)$



[4 pts.] **b)** $\frac{1}{2}(1 + u(t))\cos(2\pi t)$ $t<0: \frac{1}{2}\cos(2\pi t)$, $t>0: \cos(2\pi t)$



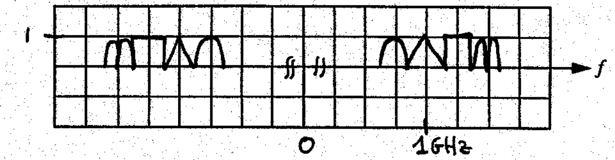
[2 pts.] **c)** What is the message $m(t)$ in the signals above? What is the modulation method (e.g., USB, etc.)?

i) $\cos[2\pi t + 2\pi t u(t)]$: $m(t)$ = $t\,u(t)$ or $u(t)$ method = PM / FM

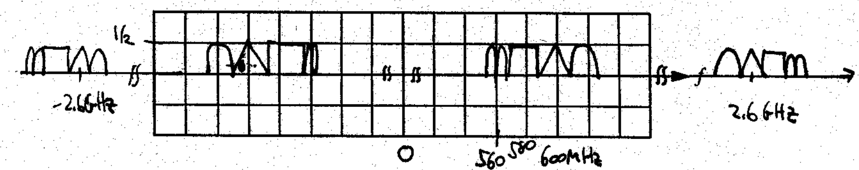
ii) $\frac{1}{2}(1 + u(t))\cos(2\pi t)$: $m(t)$ = $u(t)$ method = AM-DSB-LC

Sketch $X_2(f)$, $Y(f)$, $Z(f)$, $Z_2(f)$, **labelling important amplitudes and frequencies.**

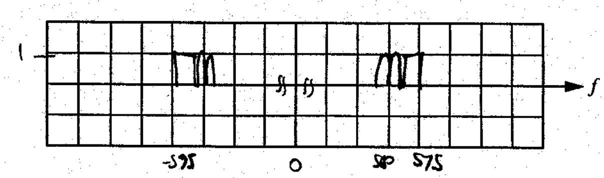
$X_2(f)$:



$Y(f)$:



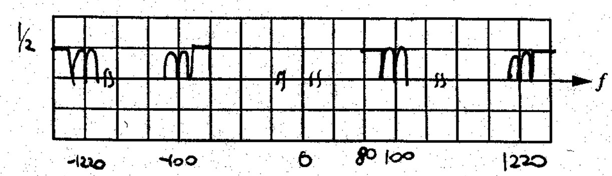
$Z(f)$:



Find f_S **such that** A_f **is one of the 20 MHz segments (half circle, triangle, pulse, or arches).**

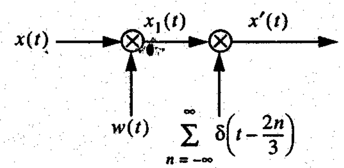
$\boxed{f_S = \frac{660 \text{ or } 460}{\text{MHz}}}$

$Z_2(f)$: for $f_S = 660$



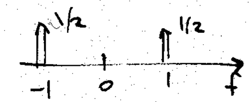
Problem 3 (25 points)

A system is described by the following block diagram:



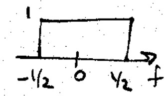
$$w(t) = \frac{\sin \pi t}{\pi t}$$

$X(f)$



$\sum \delta(t - \frac{2n}{3}) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{3}{2} \sum \delta(f - \frac{3}{2}k)$

$W(f) =$

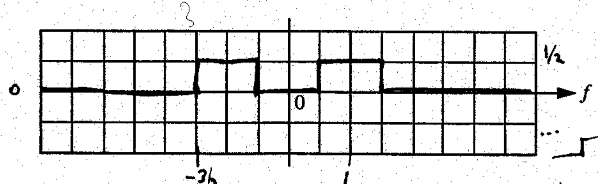


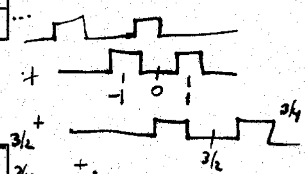
[8 pts.] **a)** Is the system with input $x(t)$ and output $x'(t)$ (circle correct answer):

causal?	YES	NO	NOT DECIDABLE
linear?	YES	NO	NOT DECIDABLE
time-invariant?	YES	NO	NOT DECIDABLE
BIBO stable?	YES	NO	NOT DECIDABLE

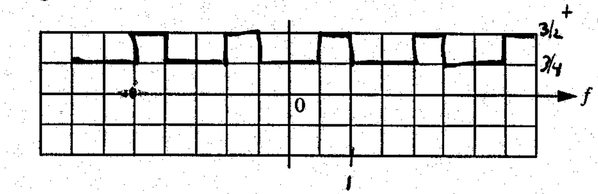
[17 pts.] **b)** For $x(t) = \cos 2\pi t$, sketch the spectra $X_1(f)$ and $X'(f)$, labelling important amplitudes and frequencies.

$X_1(f)$





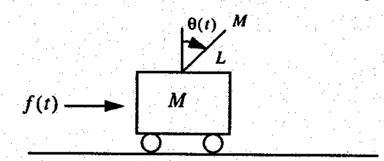
$X'(f) = X_1(f) * \frac{3}{2} \sum \delta(f - \frac{3}{2}k)$



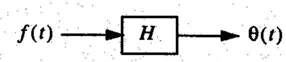
Problem 4 (25 points)

The differential equation for a broom balancing system below is given by:

$$\ddot{\theta}(t) = \theta(t) - f(t)$$



in terms of block diagrams, and with zero initial conditions;



$s^2\theta(s) = \theta(s) - F(s)$

$\theta(s)[s^2 - 1] = -F(s)$

[5 pts.] **a)** Find $H(s)$. $H(s) = \frac{\theta(s)}{F(s)} = \boxed{-1/(s^2-1)}$

[10 pts.] **b)** Suppose we measure $\theta(t)$ and use a feedback of the form $f(t) = \alpha\theta(t)$ for $\dot{\theta}(0^-) = 0$ and $\theta(0^-) = \frac{1}{2}$. Find α such that $\theta(t) = \frac{1}{2}e^{-10t}u(t)$. If no such α exists, explain why.

$\ddot{\theta}(t) = \theta(t) - \alpha(\theta(t))$

$s^2\theta(s) - s\theta(0^-) - \dot{\theta}(0^-) = (1-\alpha)\theta(s)$

$\theta(s)\,(s^2 + (\alpha - 1)) = s\theta(0^-) + \dot{\theta}(0^-) = \frac{1}{2}s$

$\theta(s) = \frac{1}{2}\frac{s}{s^2 + (\alpha-1)}$. roots of denominator = poles

$s = \pm\sqrt{\alpha - 1}$

Since if $\alpha - 1 > 0$ we get one pole in RHP $\Rightarrow$ unstable
or if $\alpha - 1 < 0$ " " two poles on $j\omega$ axis $\Rightarrow$ marginally stable
} no such α.

[10 pts.] **c)** Using a feedback of the form $f(t) = 11\theta(t) + \beta\dot{\theta}(t)$ and the same values from (b), above, find β such that $\theta(t) \to 0$ as $t \to \infty$. Justify your answer.

$s^2\theta(s) - s\theta(0^-) - \dot{\theta}(0^-) = \theta(s) - [11\theta(s) + \beta s\theta(s) - \beta\theta(0^-)]$

$\theta(s)[s^2 + \beta s + 10] = s\theta(0^-) - \dot{\theta}(0^-) + \beta\theta(0^-) = [s + \beta]\frac{1}{2}$

$\theta(s) = \frac{1}{2}\frac{s+\beta}{s^2 + \beta s + 10}$ poles at $s = \frac{-\beta \pm \sqrt{\beta^2 - 40}}{2}$

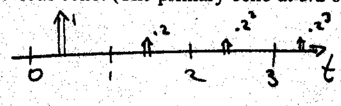
choose $\beta > 0$. for poles in LHP.

Find value $\lim_{s \to 0} s\theta(s) = 0$ ✓

Problem 5 (35 points)

You wish to determine the impulse response of an auditorium, so that you can remove the echoes from a signal recorded in that room.

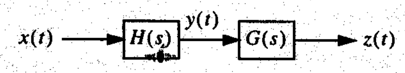
From measurements made in the auditorium, you know that if you generate an impulse at $t = 0$s in one location, you hear that impulse at $t = \frac{1}{2}s$ at your location, and echoes at 1s intervals with primary echo at $t = \frac{3}{2}s$. The amplitude of each successive echo is 20% that of the previous echo. (The primary echo at 3/2 sec has an amplitude of 20% of the direct impulse at $t = \frac{1}{2}$ sec.)



[10 pts.] a) What is $h(t)$? $\sum_{n=0}^{\infty} (.2)^n \delta(t - \frac{1}{2} - n)$

[5 pts.] b) What is $H(s)$? $\sum_{n=0}^{\infty} (.2)^n e^{-s/2} e^{-sn}$, for $Re(s) > 0$, $H(s) = \frac{e^{-s/2}}{1-0.2e^{-s}}$

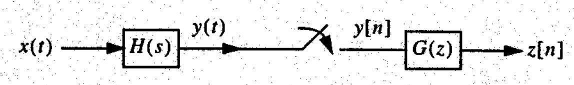
[10 pts.] c) With $H(s)$ as above,



Design $G(s)$ such that $z(t) = x(t)$. What is $g(t)$?

$G(s) = \frac{1-.2e^{-s}}{e^{-s/2}} = e^{s/2} - .2e^{-s/2}$	$g(t) = \delta(t+\frac{1}{2}) - 0.2\delta(t-\frac{1}{2})$

[10 pts.] d) With $H(s)$ as above,



$t = nT$
$T = \frac{1}{4}s$

Design $G(z)$ such that $z[n] = x(t)|_{t=nT}$. The switch represents sampling $y(t)$ such that $y[n] = y(nT)$. Use impulse invariance techniques to find $h[n]$ and $H(z)$.

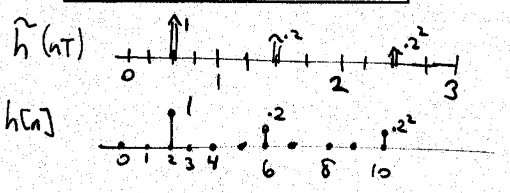
$$\boxed{G(z) = \frac{1}{H(z)} = z^2(1-.2z^{-4})}$$

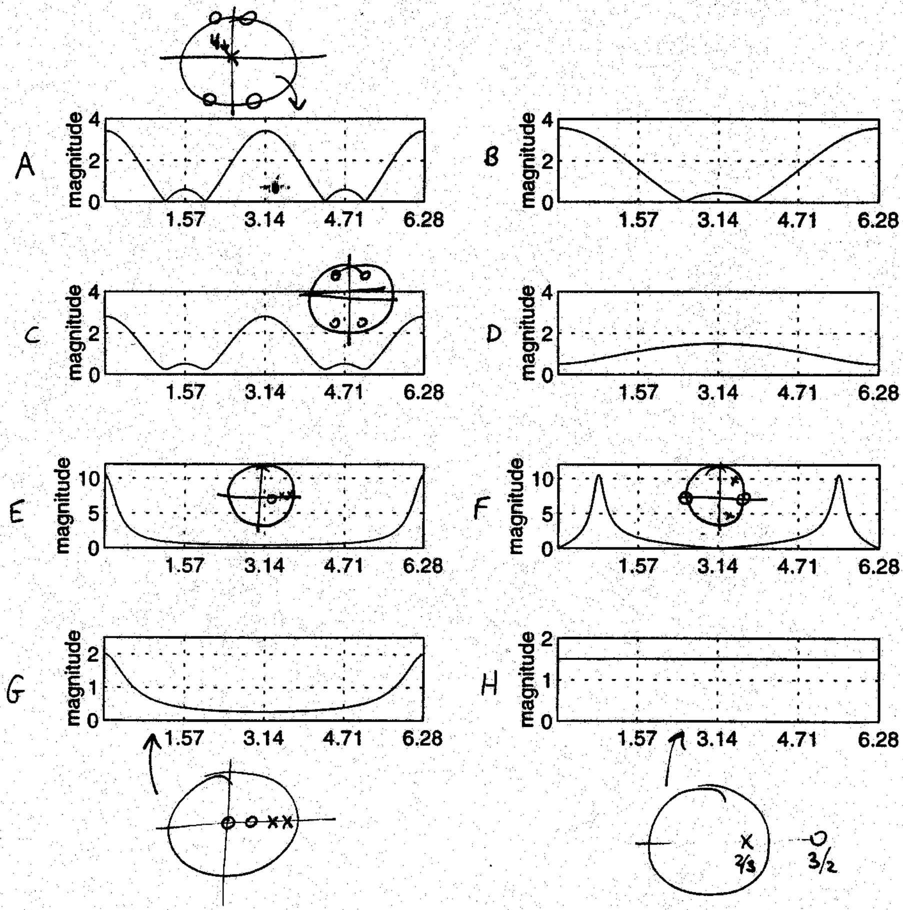
$h[n] = \sum_{k=0}^{\infty} (.2)^k \delta[n-2-4k]$

$H(z) = \sum_{k=0}^{\infty} (.2^k) z^{-2} z^{-4k}$

$= z^{-2} \sum \left(\frac{.2}{z^4}\right)^k$

$= \frac{z^{-2}}{1-.2z^{-4}}$





Problem 7 (22 points)

$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$h[n]$ is unit sample response of a linear shift invariant system.

$\frac{1}{1-\frac{4}{5}z^{-1}} \cdot \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{8/3}{1-4/5z^{-1}} + \frac{-5/3}{1-1/2z^{-1}}$

[3 pts.] a) If $x[n] = (4/5)^n u[n]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n]$, find $y[n]$.

$\boxed{y[n] = \left[\frac{8}{3}(4/5)^n - \frac{5}{3}(1/2)^n\right] u[n]}$

[3 pts.] b) If $x[n] = e^{j(\pi/4)n}$ and $h[n] = \delta[n] + \delta[n-2]$, find $y[n]$.

$\boxed{y[n] = (e^{j\pi/4})^n + (e^{j\pi/4})^{n-2}}$

[3 pts.] c) If $x[n] = e^{jn\frac{\pi}{4}}$ and $Z\{h[n]\} = H(z) = \frac{z-3/2}{z-2/3}$, find $y[n]$.

$\boxed{y[n] = e^{jn\pi/4}\left(\frac{e^{j\pi/4}-3/2}{e^{j\pi/4}-2/3}\right)}$

[3 pts.] d) If $x[n] = (4/5)^n u[n]$ and $h[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{4}\delta[n-2]$, find $y[n]$.

$\boxed{y[n] = \left(\frac{4}{5}\right)^n u[n] + \frac{1}{2}\left(\frac{4}{5}\right)^{n-1} u[n-1] +} \frac{1}{4}\left(\frac{4}{5}\right)^{n-2} u[n-2]$

[3 pts.] e) If $x[n] = \delta[n]$ and the DTFT of $h[n] = H(e^{j\omega T}) = 1 + e^{-4j\omega T}$, find $y[n]$.

$\boxed{y[n] = \delta[n] + \delta[n-4]}$

$H(z) = 1 + z^{-4}$

$h[n] = 1 + \delta[n-4]$

[4 pts.] f) Is $H(z) = \frac{1}{1-2z^{-1}}$, the transfer function for a BIBO stable system? Why or why not?

$H(z) = \frac{z}{z-2}$

pole at $z=2$.

for causal system to be BIBO, $|z| < 1$

[3 pts.] g) If $H(z) = \frac{a-bz^{-1}}{c-dz^{-1}}$, what is the corresponding linear difference equation with input $x[n]$ and output $y[n]$?

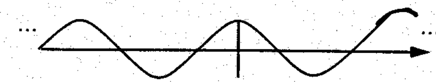
$\boxed{c\,y[n] - d\,y[n-1] = a\,x[n] - b\,x[n-1]}$

Problem 8 (40 points)

Given $x(t)$ or $x[n]$, match its FT or DFT, as appropriate, with corresponding letter from next page.

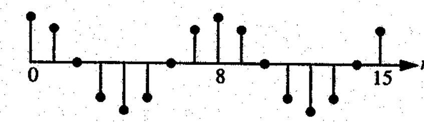
Note: Assume all $x[n]$ are periodic with period 16; $x_1(t)$ and $x_6(t)$ are periodic.

$x_1(t)$



$X_1(f)$ is sketch $\boxed{A}$

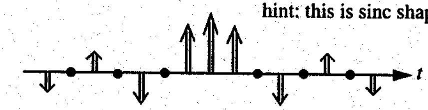
$x_2[n]$



$X_2[k]$ is sketch $\boxed{Q}$

$x_3(t)$

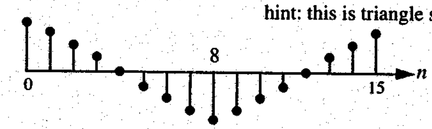
hint: this is sinc shaped



$X_3(f)$ is sketch $\boxed{G}$

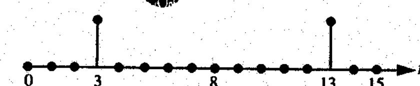
$x_4[n]$

hint: this is triangle shaped



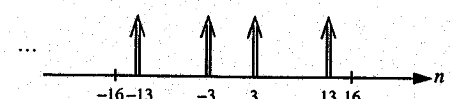
$X_4[k]$ is sketch $\boxed{T}$

$x_5[n]$



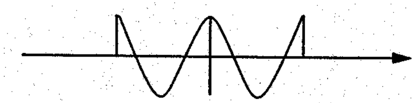
$X_5[k]$ is sketch $\boxed{N}$

$x_6(t)$



$X_6(f)$ is sketch $\boxed{F}$

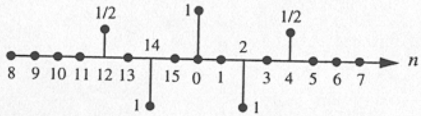
$x_7(t)$



$X_7(f)$ is sketch $\boxed{I}$

hint: assume $\pi(t) = \begin{cases} 0 & t < -1/2, t > 1/2 \\ \frac{1}{2} & t = -1/2, t = 1/2 \\ 1 & -1/2 < t < 1/2 \end{cases}$

$x_8[n]$



$X_8[k]$ is sketch $\boxed{K}$

or equivalently: