Problem 1 (20 points)

Classify the following systems. In each column, write "yes", "no", or "?" (use "?" if not decidable with given information). The input to the system is $x(t)$ and output is $y(t)$. (To discourage random guessing, +1 for correct, 0 for blank, $-\frac{1}{2}$ for incorrect.)

System	Causal	Linear	Time-invariant	BIBO stable
a. $y(t) = x(t) + u(t-1)$	YES	NO	NO	YES
b. $y(t) = \int_{-\infty}^{t} \sqrt{\lvert x(\tau)\rvert}\, d\tau$	YES	NO	YES	NO
c. $y(t) = x(t) * (\sin(\omega_0 t) u(t))$	YES	YES	YES	NO
d. $y(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$	NO	YES	YES	NO
e. $y(t) = x(t) + \int_{-\infty}^{\infty} x(\tau)[e^{-(t-\tau)} u(t-\tau)]\, d\tau$	YES	YES	YES	YES

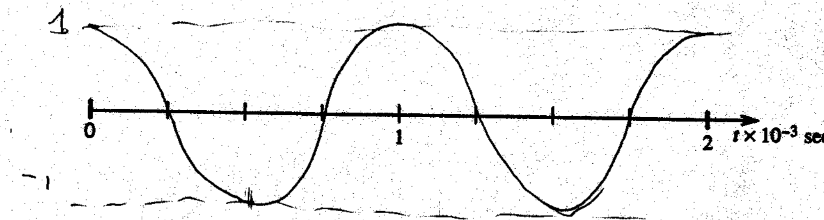
$= x(t) * [e^{-t} u(t)]$

so $y(t) = x(t) * \underbrace{[\delta(t) + e^{-t} u(t)]}_{h(t)}$

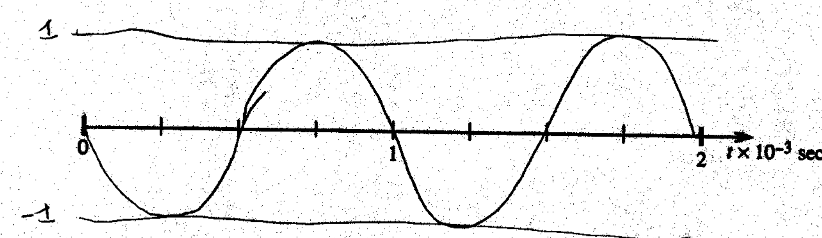
Problem 2 (15 points)

Sketch the following time functions, labelling maximum amplitude.

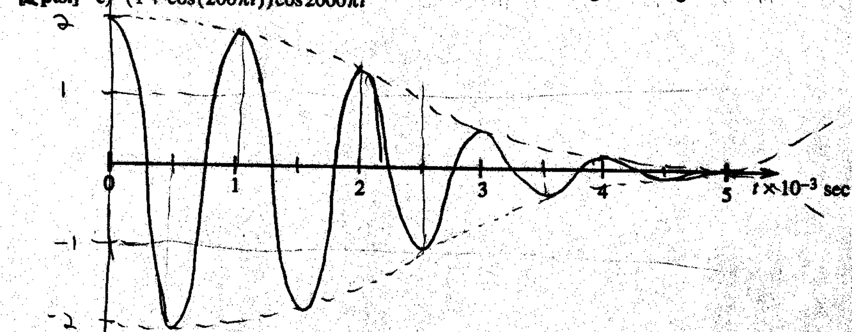
[1 pts.] a) $\cos(2000\pi t)$ $\quad T_0 = \frac{2\pi}{2000\pi} = 10^{-3}\ s$



[1 pts.] b) $\cos(2000\pi t + \pi/2) \quad = -\sin(2000\pi t)$

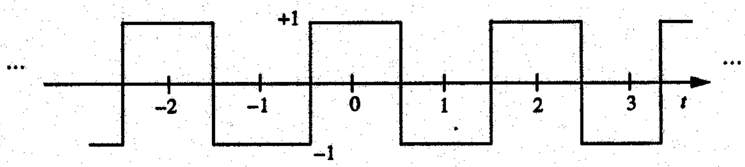


[1 pts.] c) $(1 + \cos(200\pi t))\cos 2000\pi t \quad \cos(200\pi t) \longrightarrow T_0 = 10^{-2}\ s$



Problem 3 (~~12~~ 30 points)

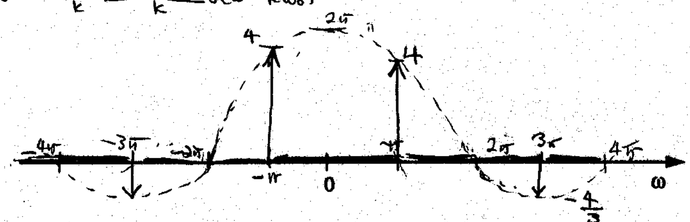
Consider a periodic signal $m(t)$ shown below:



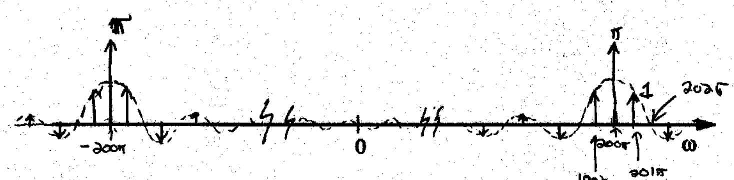
From Table 4.3, $m(t) = -1 + 2$ periodic square wave w/ $T_1 = \frac{1}{2}$, $T_0 = 2$, $\omega_0 = \pi$

12 [~~0~~ pts.] **a)** Sketch $M(\omega)$, labelling heights, center frequency, and spacing.

$M(\omega) = -2\pi\delta(\omega) + 2\sum_k \frac{2\sin k2\pi/4}{k}\delta(\omega - k\omega_0)$



10 [~~0~~ pts.] **b)** Let $x(t) = (1 + 0.5m(t))\cos\omega_c t$, with $\omega_c = 200\pi$. Sketch $X(\omega)$, labelling heights, center frequencies, and spacing. $X(\omega) = \pi[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)] + \frac{1}{4}[M(\omega-\omega_c) + M(\omega+\omega_c)]$



[2 pts.] **c)** What kind of modulation technique is used to generate $x(t)$? (Circle one.)

AM DSB — ?? with carrier	upper sideband	
AM DSB — no carrier	lower sideband	
wideband frequency modulation	no wideband phase modulation	interrupted continuous wave
narrowband frequency modulation	narrowband phase modulation	

[0 pts.] **d)** What is the ratio of power in the sidebands to power in the carrier for $x(t)$?

THE POWER IN THE CARRIER CORRESPONDS TO THE SIGNAL $\cos(200\pi t)$, WHICH HAS POWER $= \sum_k |a_k|^2 = |\frac{\pi}{2\pi}|^2 + |\frac{\pi}{2\pi}|^2 = \frac{1}{2}$ (PARSEVALS)

THE SIDEBANDS CORRESPOND TO A MODULATED SQUARE WAVE — BECAUSE IT IS MODULATED WE GET 2 COPIES, A SINGLE COPY IS JUST $\frac{M(\omega+200\pi)}{4}$. THE SHIFTING OF $M(\omega)$ BY 200π DOESN'T AFFECT ITS POWER.

F.S. OF $M(\omega)$

SO POWER $\{M(\omega - 200\pi)\}$ = POWER $\{M(\omega)\} = \sum_k |a_k|^2 = \frac{1}{T_0}\int_{T_0} |m(t)|^2 dt$

$= \frac{1}{2}\int_{-1}^{1} |1|^2 dt = \frac{2}{2} = 1$

THUS POWER $\{\frac{M(\omega)}{4}\} = \frac{1}{4}$, AND SINCE WE GOT 2 COPIES BY MODULATING, TOTAL SIDEBAND POWER $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

RATIO OF $\frac{P_{SIDEBANDS}}{P_{CARRIER}} = \frac{\frac{1}{2}}{\frac{1}{2}} = \boxed{1/2}$

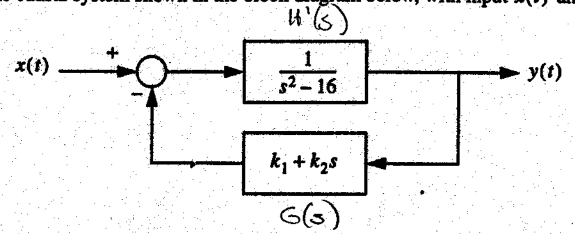
[0 pts.] **e)** Explain (assuming knowledge of EE40) what modulation is good for and why it is used in communications systems.

MODULATION IS GOOD FOR COMBINING DIFFERENT SIGNALS TOGETHER INTO 1 SIGNAL IN A SUCH A WAY THAT WE CAN RECOVER THE INDIVIDUAL SIGNALS EASILY. WE CAN DO THIS USING CIRCUITS & FILTERS.

MODULATION IS USED IN COMMUNICATIONS TO TRANSMIT MULTIPLE SIGNALS THAT HAVE THE SAME BANDWIDTH (ex. $[-B$ TO $+B]$) BY MOVING EACH SIGNAL'S SPECTRUM INTO NON-OVERLAPPING AREAS IN FREQUENCY. ALSO, HIGHER FREQUENCY SIGNALS ARE LESS SUSCEPTIBLE TO ELECTRO-MAGNETIC INTERFERENCE.

Problem 4 ~~(12 points)~~ 25

Consider the causal system shown in the block diagram below, with input $x(t)$ and output $y(t)$:



~~[0 pts.]~~ 7 a) Find the transfer function for the system.

$$H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{1}{s^2+k_2 s-16+k_1}}$$

$$H(s) = \frac{H'(s)}{1+G(s)H'(s)} = \frac{\frac{1}{s^2-16}}{1+\frac{k_1+k_2 s}{s^2-16}} = \frac{1}{s^2+k_2 s-16+k_1}$$

~~[0 pts.]~~ 6 b) With $k_1 = 0, k_2 = 0$, what is the impulse response for the system? Is the system BIBO stable? Why or why not?

$k_1 = 0, k_2 = 0 \Rightarrow$ NO FEEDBACK

$H(s) = H'(s) = \frac{1}{s^2-16}$ POLES AT $s = +/- 4$

$H(s) = \frac{1}{(s+4)(s-4)} = \frac{-\frac{1}{8}}{s+4} + \frac{\frac{1}{8}}{s-4}$

$h(t) = -\frac{1}{8}e^{-4t}u(t) + \frac{1}{8}e^{4t}u(t)$

$$h(t) = \boxed{\frac{1}{8}u(t)\left[-e^{-4t}+e^{4t}\right]}$$

~~[0 pts.]~~ 6 c) Find values of k_1 and k_2 such that the closed loop system has 2 poles at $s = -4$.

2 poles at $s = -4 \Rightarrow H(s) = \frac{1}{(s+4)^2} = \frac{1}{s^2+8s+16}$

$-16 + k_1 = 16 \quad k_1 = 32$

$k_2 s = 8s \quad k_2 = 8$

$\boxed{k_1 = 32}$ $\boxed{k_2 = 8}$

~~[0 pts.]~~ 6 d) For positive k_2, what is the minimum value of k_1 for the closed loop system to be BIBO stable?

$\boxed{k_1 > 16}$

$\frac{1}{s^2+k_2 s-16+k_1}$ HAS POLES AT $s_p = \frac{-k_2 \pm \sqrt{k_2^2-4(k_1-16)}}{2}$

FOR POLES IN LHP, $\text{RE}\{s_p\} < 0 \Rightarrow \sqrt{k_2^2-4(k_1-16)} < k_2$

$\Rightarrow \cancel{k_2^2} + 64 - 4k_1 < \cancel{k_2^2}$

$k_1 > 16$

Problem 5 ~~(12 points)~~ 25

10:30 → 10:40

Consider the following difference equation:

$$y[n] - \frac{3}{8}y[n-1] + \frac{1}{8}y[n-2] = 5x[n]$$

$y[n-1] \longrightarrow y[-1] + z^{-1}Y(z)$

$y[n-2] \longrightarrow y[-2] + z^{-1}y[-1] + z^{-2}Y(z)$

$Y(z)\left(1 - \frac{3}{8}z^{-1} + \frac{1}{8}z^{-2}\right) = 5X(z)$

$-\frac{3}{8}y[-1]$

$+\frac{1}{8}\left[y[-2] + z^{-1}y[-1]\right]$

~~[0 pts.]~~ 5 a) For $x[n] = \delta[n-1]$, what is the zero state response?

$X(z) = z^{-1}$

$$\boxed{y_{ZSR}[n] = 40\left[\left(\tfrac{1}{4}\right)^n - \left(\tfrac{1}{8}\right)^n\right]u[n]}$$

y_{ZSR}

$Y(z) = \frac{5z^{-1}}{1-\frac{3}{8}z^{-1}+\frac{1}{8}z^{-2}} = \frac{40}{1-\frac{1}{4}z^{-1}} + \frac{-40}{1-\frac{1}{8}z^{-1}}$

~~[0 pts.]~~ 5 b) Assuming $y[-1] = 1$ and $y[-2] = 2$, what is the zero input response?

$$\boxed{y_{ZIR}[n] = -\tfrac{31}{4}\left(\tfrac{1}{4}\right)^n u[n] + \tfrac{63}{8}\left(\tfrac{1}{8}\right)^n u[n]}$$

$Y(z) = \frac{3/8 - 1/8(2+z^{-1})}{1-\frac{3}{8}z^{-1}+\frac{1}{8}z^{-2}} = \frac{\frac{1}{8}-z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{8}z^{-1})} = \frac{-31/4}{1-\frac{1}{4}z^{-1}} + \frac{63/8}{1-\frac{1}{8}z^{-1}}$

~~[0 pts.]~~ 5 c) Is the system BIBO stable? Why or why not?

Yes. poles at $z = 1/4,\ z = 1/8$

are inside unit circle.

~~[0 pts.]~~ 5 d) What is $\lim_{n\to\infty} y[n]$ if $x[n] = 3u[n]$?

$$\boxed{\lim_{n\to\infty} y[n] = 60/3}$$

$= \lim_{z\to 1} \cancel{(1-z^{-1})}3\,\frac{1}{\cancel{1-z^{-1}}}\,\frac{5}{1-\frac{3}{8}z^{-1}+\frac{1}{8}z^{-2}} = \frac{15}{1-\frac{3}{8}+\frac{1}{8}} = \frac{15}{3/4} = 60/3$

~~[0 pts.]~~ 5 e) Assuming zero initial conditions, what is the steady state response to $x[n] = e^{j\pi n}$?

$$\boxed{y[n] = \tfrac{10}{3}\ e^{j\pi n}}$$

$e^{j\pi n} * h[n] = e^{j\pi n} H(e^{j\pi})$

$= \sum_k h[k]e^{j\pi(n-k)} = e^{j\pi n}\underbrace{\sum_k h[k]e^{-j\pi k}}$

$H(z) = \frac{5}{1-\frac{3}{8}z^{-1}+\frac{1}{8}z^{-2}}\Big|_{z=e^{j\pi}=-1} = \frac{5}{1-(3/8)(-1)+1/8(-1)^2} = \frac{5}{3/2} = \frac{10}{3}$

$1 + 3/8 + 1/8 = 3/2$

Problem 6 (?? points) 25   10:40 → 10:50

A continuous time filter has impulse response

$$h(t) = (e^{-t} - e^{-.5t})u(t)$$

$e^{-t} - e^{-.5t}$

[? pts.] a) Find the corresponding digital filter $H(z)$ using impulse invariant techniques and sample time $T = 0.5$ sec where ($e^{-.5} \approx .61$ and $e^{-.25} \approx .78$).

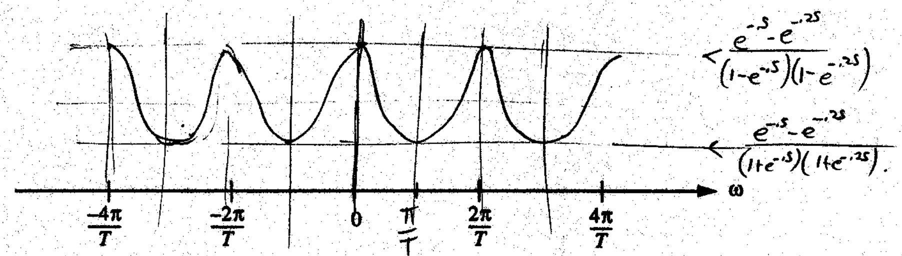
$$h[n] = \left(e^{-0.5n} - e^{-.25n}\right)u[n]$$

$$= \left((e^{-.5})^n - (e^{-.25})^n\right)u[n]$$

$$H(z) = \frac{1}{1-e^{-.5}z^{-1}} - \frac{1}{1-e^{-.25}z^{-1}}$$

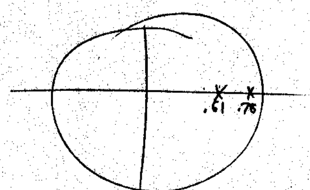
$$\boxed{H(z) = \frac{1}{1-e^{-.5}z^{-1}} - \frac{1}{1-e^{-.25}z^{-1}}}$$

[? pts.] b) Sketch $|H(e^{j\omega T})|$ in range $\frac{-4\pi}{T} < \omega < \frac{4\pi}{T}$, labelling maximum and minimum amplitude. (Maximum and minimum may be left as functions of e^x). with $T = 0.5$ sec.



$$H(z) = \frac{(1-e^{-.25}z^{-1}) - (1-e^{-.5}z^{-1})}{(1-\alpha z^{-1})(1-\beta z^{-1})} = \frac{z^{-1}\left(e^{-.5} - e^{-.25}\right)}{(1-\alpha z^{-1})(1-\beta z^{-1})}$$

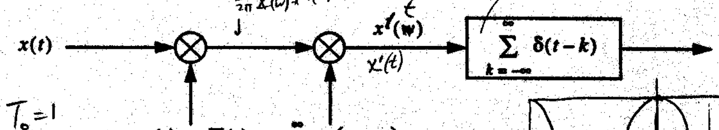
$$= \frac{z\left(e^{-.5} - e^{-.25}\right)}{(z-\alpha)(z-\beta)}$$



Problem 7 (?? points) 40 (challenge problem) 10:47 → 11:20

A system is described by the following block diagram:

$2\pi \sum \delta(\omega - k2\pi)$

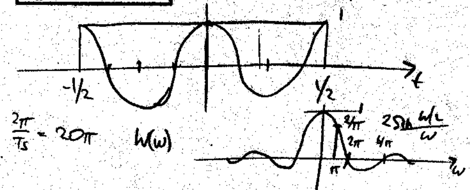


$T_0 = 1$

double u

$W(\omega) = \frac{2\sin\omega/2}{\omega}$

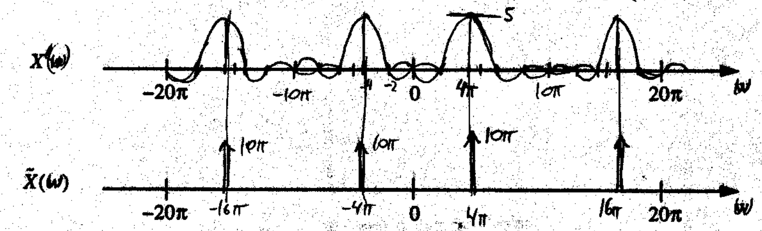
$T_s = 1/10$, $\frac{2\pi}{T_s} = 20\pi$



all omega

[? pts.] a) Let $x(t) = \cos 4\pi t$. Sketch $X'(\omega)$ and $\tilde{X}(\omega)$, labelling peak magnitude, zero crossing(s), and spacing. (Hint: $X'(\omega)$ and $\tilde{X}(\omega)$ should be real.)

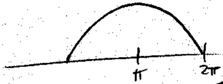
$$X'(\omega) = \left[\frac{\sin\frac{(\omega-4\pi)}{2}}{(\omega-4\pi)} + \frac{\sin(\omega+4\pi)/2}{\omega+4\pi}\right] * \frac{1}{T_s}\sum\delta\left(\omega - \frac{k\pi}{T_s}\right)$$



Problem 8 (3 points) 1120 → 1135

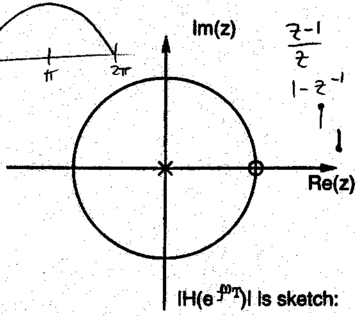
~~coeff. of highest power numerator~~

For each pole–zero diagram below, fill in the box with the letter of the corresponding frequency response and unit sample response from the next pages. All diagrams represent causal systems.



$\frac{z-1}{z}$

$1-z^{-1}$



$|H(e^{j\Omega T})|$ is sketch: $\boxed{E}$

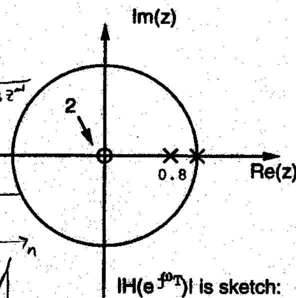
h[n] is sketch: $\boxed{M}$

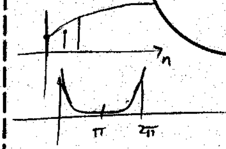
$5u[n]-4(.8)^n u[n]$

$\frac{z^2}{(z-1)(z-.8)}$

$\frac{5}{1-z^{-1}} + \frac{-4}{1-.8z^{-1}}$

$\frac{1}{.8} = \frac{5}{4}$





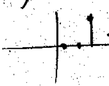
$|H(e^{j\Omega T})|$ is sketch: $\boxed{A}$

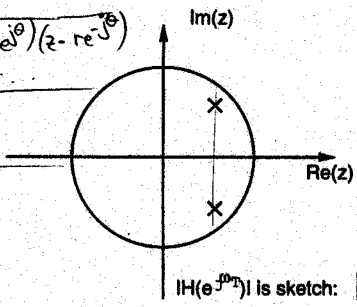
h[n] is sketch: $\boxed{L}$

$\frac{1}{(z-re^{j\theta})(z-re^{-j\theta})}$

z^{-2} . . .

$z^2 \overline{)\;1\;}$



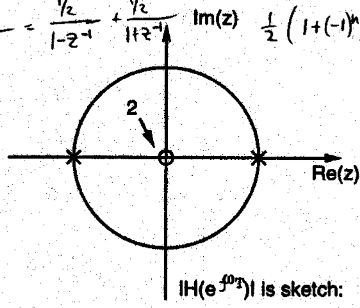


$|H(e^{j\Omega T})|$ is sketch: $\boxed{B}$

h[n] is sketch: $\boxed{K}$

$\frac{z^2}{z^2-1} = \frac{1/2}{1-z^{-1}} + \frac{1/2}{1+z^{-1}}$

$\frac{1}{2}\left(1+(-1)^n\right)u[n]$

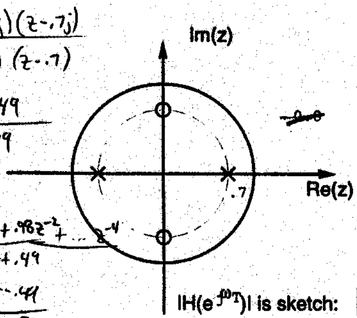


$|H(e^{j\Omega T})|$ is sketch: $\boxed{C}$

h[n] is sketch: $\boxed{J}$

$\frac{(z+.7j)(z-.7j)}{(z+.7)(z-.7)}$

$= \frac{z^2+.49}{z^2-.49}$



$1+.98z^{-2}+\ldots z^{-4}$

$z^2-.49 \overline{)\,z^2+.49}$

$z^2-.49$

$.98$

$.98 - (.49)(.98)$

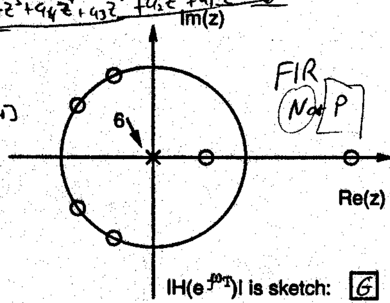
$+.4$

$|H(e^{j\Omega T})|$ is sketch: $\boxed{D}$

h[n] is sketch: $\boxed{I}$

$\frac{z^6+a_5z^5+a_4z^4+a_3z^3+a_2z^2+a_1z^1+a_0}{z^6}$

$\delta[n]$ $+a_5\delta[n-1]$



FIR

Not P

$|H(e^{j\Omega T})|$ is sketch: $\boxed{G}$

h[n] is sketch: $\boxed{N}$