Problem 1 (20 points)
Classify the following systems. In each column, write "yes", "no", or "?" (use "?" if not decidable with given information). The input to the system is \( x(t) \) and output is \( y(t) \). (To discourage random guessing, +1 for correct, 0 for blank, -1 for incorrect.)

<table>
<thead>
<tr>
<th>System</th>
<th>Causal</th>
<th>Linear</th>
<th>Time-invariant</th>
<th>BIBO stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( y(t) = x(t) + u(t-1) )</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>b. ( y(t) = \int u(t) dt )</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>c. ( y(t) = x(t) \ast (\sin(\omega_0 t) u(t)) )</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>d. ( y(t) = \int_{t-1}^{t} x(t) dt )</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>e. ( y(t) = x(t) + \int_{t-1}^{t} x(t) [e^{-t-t_0} u(t-t)] dt )</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

\[
= x(t) \ast [e^{-t} u(t)]
\]

So \( y(t) = x(t) \ast [s(t) + e^{-t} u(t)] \)

Problem 2 (18 points)
Sketch the following time functions, labelling maximum amplitude.

[a. pts.] a) \( \cos(2000\pi t) \)
\[
T_0 = \frac{2\pi}{2000\pi} = 10^{-3} s
\]

[b. pts.] b) \( \cos(2000\pi t + \pi/2) = -\sin(2000\pi t) \)

[c. pts.] c) \( (1 + \cos(200\pi t)) \cos(200\pi t) \)
\[
\cos(200\pi t) \rightarrow T_0 = 10^{-3} s
\]
Consider a periodic signal $m(t)$ shown below:

(0 pts.) a) Sketch $M(\omega)$, labelling heights, center frequency, and spacing. 

(12 pts.) b) From Table 4.3, $w(t) = 1 + 2A \cos(2\pi f_0 t)$ where $w(t) = 2A \cos(2\pi f_0 t)

(0 pts.) c) What kind of modulation technique is used to generate $x(t)$? (Circle one.)

AM DSB — ?? with carrier
AM DSB — no carrier
Wideband frequency modulation
Wideband phase modulation
Interrupted continuous wave
Narrowband frequency modulation
Narrowband phase modulation

[6 pts.] d) What is the ratio of power in the sidebands to power in the carrier for $x(t)$?

Power in the carrier corresponds to the signal $\cos(\omega_0 t)$, which has

$$P_{\text{carrier}} = \frac{\text{E}_p}{2} = \frac{\text{E}_p}{2}$$

The sidebands correspond to a modulated square wave because it is modulated

To convey a single pair is just $w(t) = 2A \cos(2\pi f_0 t)$, the shifting of $w(t)$ by $200\text{Hz}$ doesn't affect its power.

So power $\sum_{k=-\infty}^{\infty} P_k = \frac{\text{E}_p}{2}$ E.S. of $H(\omega)$

Thus power $\sum_{k=-\infty}^{\infty} P_k = \frac{A^2}{4}$, and since we got 2 cosines by modulating,

Total Sideband Power $= \frac{A^2}{2}$

Ratio of $\frac{P_{\text{sidebands}}}{P_{\text{carrier}}}$ = $\frac{\frac{A^2}{4}}{\frac{A^2}{2}} = \frac{1}{2}$

[6 pts.] e) Explain (assuming knowledge of EE40) what modulation is good for and why it is used in communications systems.

Modulation is good for combining different signals together into a signal in a way that we can recover the individual signals easily. We can do this using circuits & filters.

Multiple modulation is used in communications to transmit signals that have the same bandwidth (e.g., [-B to +B]) by moving each signal's spectrum into non-overlapping areas in frequency. Also, higher frequency signals are less susceptible to electro-magnetic interference.
Problem 4 (12 points)
Consider the causal system shown in the block diagram below, with input $x(t)$ and output $y(t)$:

$$H(s) = \frac{1}{s^2 - 16}$$

[8 pts.] a) Find the transfer function for the system.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 4s + 16}$$

b) With $k_1 = 0$, $k_2 = 0$, what is the impulse response for the system? Is the system BIBO stable? Why or why not?

$$h(t) = \frac{1}{2} e^{-4t} u(t)$$

[6 pts.] c) Find values of $k_1$ and $k_2$ such that the closed-loop system has 2 poles at $s = -4$.

$$k_1 = 32$$
$$k_2 = 8$$

[8 pts.] d) For positive $k_1$, what is the minimum value of $k_2$ for the closed-loop system to be BIBO stable?

$$k_2 > 16$$
Problem 6 (12 points)
Continuous-time filter has impulse response
\[ h(t) = (e^{-5t})u(t) \]
\[ e^{-t} - e^{-2t} \]

\[ H(z) = \frac{1}{1 - e^{-5z^{-1}} - e^{-2z^{-1}}} \]

(a) Find the corresponding digital filter \( H(z) \) using impulse invariant techniques and sample time \( T = 0.5 \) sec where \( e^{-2s} = 0.61 \) and \( e^{-2s} = 0.75 \).

(b) Sketch \( |H(e^{j\omega T})| \) in range \(-\frac{1}{2T} < \omega < \frac{1}{2T}\), labelling maximum and minimum amplitude. (Maximum and minimum may be left as functions of \( e^{\omega T} \).) \( \omega_{dc} \quad T = 0.5 \text{ sec} \).

\[ H(z) = \frac{(1 - e^{-5z^{-1}} - e^{-2z^{-1}})}{(1 - e^{-5z^{-1}})(1 - e^{-2z^{-1}})} \]

\[ \phi(\omega) = \frac{2\sin(\frac{\omega}{2})}{\omega} \]

Problem 7 (10 points)
A system is described by the following block diagram:

\[ x(t) \rightarrow \ast \rightarrow x(t) \rightarrow \sum \delta(t-k) \rightarrow \dot{x}(t) \]

\[ T_0 = \frac{1}{10} \]

\[ w(t) = \Pi(t) \sum \delta(t-n) \]

[10 pts.] a) Let \( x(t) = \cos(4\pi t) \). Sketch \( X(\omega) \) and \( \tilde{X}(\omega) \), labelling peak magnitude, zero crossing(s), and spacing. (Hint: \( X(\omega) \) and \( \tilde{X}(\omega) \) should be real.)

\[ X(\omega) = \left[ \frac{\sin(\frac{\omega}{2})}{\omega} + \frac{3\sin(\omega n)}{\omega}\right] \frac{1}{\omega^4} \]

\[ \tilde{X}(\omega) \]
Problem 8 (3 points)  

For each pole-zero diagram below, fill in the box with the letter of the corresponding frequency response and unit sample response from the next pages. All diagrams represent causal systems.

\[ h[n] \text{ is sketch: } \]