EECS 120, Spring 1994
Midterm #2
Professor J. M. Kahn

Problem #1 (40 pts.)

Consider the circuit shown below. The input and output voltages are x(t) and y(t), respectively.

a. (10 pts.) Show that the input and output are related by the differential equation:

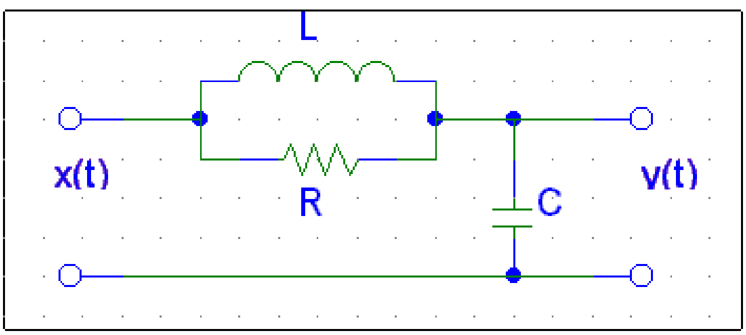
$$C\frac{d^2y}{dt^2}+\frac{1}{R}\frac{dy}{dt}+\frac{1}{L}y=\frac{1}{R}\frac{dx}{dt}+\frac{1}{L}x$$

For the remainder of the problem, assume C = 1 F, R = 1/2 Ω and L = 1/2 H

b. (10 pts.) What is the circuit's transfer function H(s)?

c. (10 pts.) Make Bode plots of the circuits's frequency response, i.e, plot as straight lines the asymptotes of 20log10 |H(jω)| and ∠H(jω) versus ω using a logarithmic scale for ω. Be sure to label the vertical and horizontal scales of your plots.

(d) (10 pts.) What is the circuit's impulse response h(t)?



Problem #2 (30 pts.)

Let h(t) = $e^{j\alpha t^2/2}$ be a "chirp" signal, so-named because its instantaneous frequency increases linearly with time.

a. (5 pts.) What is the instantaneous frequency of h(t)? (Note: For y(t) of the form y(t) = $e^{j\theta(t)}$, the instantaneous frequency is defined by $\omega_i = d\theta / dt$.)

b. (15 pts.) Show that the output is y(t) = X(αt), where X(ω) is the Fourier transform of the input. *Hint:* write out an expression for y(t), explicitly showing the convolution integral. (This method for determining the Fourier transform of x(t) is referred to as the "chirp-transform algorithm.")

c. (10 pts.) Sketch the output y(t) as a function of time if α = 2π and the input is x(t) = sin(π)/ (π). Be sure to label the axes in your plot.

Problem #3 (30 pts.)

In the system shown here, a message signal $m(t) = \cos 2\pi f_m t$ is the frequency-modulated with frequency deviation constant f_Δ onto a carrier at frequency fc. The resulting frequency-modulated signal x(t) is then squared to yield y(t) = x2(t).

a. (5 pts.) Give an expression for the FM signal x(t) in terms of f_c, f_m and the modulation index $\beta = f_\Delta / f_m$.

b. (10 pts,) give an expression for $y(t) = x^2(t)$, showing that it contains a d.c. term and another term centered at a nonzero carrier frequency. *Hint:* use the identity $\cos^2\theta = ½ [1 + \cos (2\theta)]$.

c. (5 pts.) What is the modulation index of the term centered at the nonzero carrier frequency?

d. (10 pts.) Plot Y(f), the Fourier transform of y(t). [It's easier to plot Y(f) directly, instead of the magnitude and phase . Show the component at d.c. and the first three sidebands on either side of the nonzero carrier frequency. Label all frequencies and amplitudes.