Problem #1
(20 pts.) The table below presents four systems that have input $x(t)$ and output $y(t)$. Please specify (yes or no) whether each system is linear, time-invariant, memoryless, and causal. You needn't justify your answers.

\[
y(t) = x(t) \otimes \sin(t) \ u(t)
\]
\[
y(t) = x(t) \otimes \cos(t) \ u(t)
\]
\[
y(t) = \int_{-\infty}^{t} \sqrt{x(t')} \ dt'
\]
\[
y(t) = x(2t) + 1
\]

Problem #2
(35 pts.) Consider an LTI system. When the input is $x(t) = u(t) - u(t-1)$, the output is $y(t) = e^{-t}u(t) - e^{-(t-1)}u(t-1)$.

(a) (5 pts.) What is the step response of the system?

(b) (5 pts.) What is the impulse response of the system?

(c) (5 pts.) Suppose that the input $x(t)$ is the signal shown here. What is the output $y(t)$?

(d) (5 pts.) What is the frequency response $H(w)$?

(e) (5 pts.) Plot the magnitude and phase of $H(w)$. Be sure to label the vertical and horizontal axes of your
plots.

(f) (5 pts.) Suppose the input is $x(t) = \sin(w_0 t)$, $w_0 > 0$. Find the output $y(t)$ in terms of real functions.

**Problem #3**
(35 pts.) A periodic signal $x(t)$ consists of impulses that are alternately scaled by 1 and -1.

(a) (5 pts.) Find an expression for $x(t)$ in terms of a sum of shifted and scaled impulses.

(b) (10 pts.) Find an expression for $x(t)$ in terms of an exponential Fourier series. The only variables appearing in your expression should be $t$ and $n$.

(c) (5 pts.) In part (b), are some of the Fourier series coefficients equal to zero? If the answer is yes, then say which coefficients are zero and why that is so.

(d) (10 pts.) An LTI with input $x(t)$ and output $y(t)$ is described by the differential equation:

Find the frequency response of the system, $H(w)$.

(e) (5 pts.) The periodic signal shown in the figure on the previous page is fed into this LTI system. Find an expression for the output $y(t)$. The only variables appearing in your answer should be $t$ and $n$.

**Problem #4**
(10 pts.) An LTI system has an impulse response $h(t) = \sin t \cdot u(t)$. If the input is $x(t) = t \cdot u(t)$, find an explicit expression for the output $y(t)$. Sketch $y(t)$, being sure to label the vertical and horizontal axes of your plot.

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