EE 120: Signals & Systems
Department of Electrical Engineering and Computer Sciences — Final Exam
University of California, Berkeley — May 10, 2016, 11:30am-2:30pm

Instructions:

- There are **eight** questions on this midterm. **Answer each question in the space provided.** You can use the additional blank pages at the end for scratch paper if necessary.
- We may use Gradescope for grading. **Do NOT write answers on the back of any sheet or in the additional blank pages, it will NOT be scanned or graded.**
- Each problem is worth 12 points, and you may solve the problems in any order.
- Show all work. If you are asked to prove something specific, you must give a derivation and not quote a fact from your notes sheet. Otherwise, you may freely use facts and properties derived in class; just be clear about what you are doing!
- None of the questions requires a very long answer, so avoid writing too much! Unclear or long-winded solutions may be penalized.
- You may use one double-sided sheet of notes. **No calculators are allowed** (or needed).

Your Name:

Your Student ID:

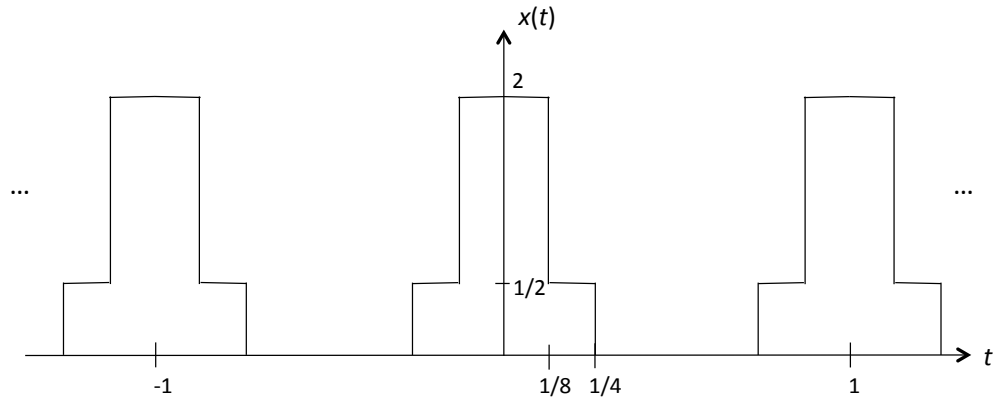
Name of Student on Your Left:

Name of Student on Your Right:

For official use – do not write below this line!

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total

Problem 1. Compute the Fourier Series coefficients of the following signal.



Problem 2. Consider the system described by the differential equation

$$y''(t) + 3y'(t) + 2y(t) = 2x'(t) + x(t).$$

You may assume the system begins at rest.

a) Find the frequency response $H(j\omega)$.

b) Find the impulse response $h(t)$.

Problem 3. The Nyquist samples of a signal $x(t)$ bandlimited to B Hz are

$$x[n] = x(nT) = \begin{cases} 1 & n = 0, 1 \\ 0 & n \neq 0, 1 \end{cases} \qquad T = \frac{1}{2B}$$

a) Give an explicit expression for $x(t)$. Simplify your answer as much as possible.

b) Sketch your signal from part a).

Problem 4. Suppose you borrow C dollars from the bank, repay in equal installments of P dollars, and that the interest rate per payment period is $\alpha \times 100\%$ on unpaid principle. Define

$$y[k] = \text{amount owed after } k\text{th payment}$$

and $x[k] = Pu[k] = \text{payment at close of } k\text{th period.}$

These quantities are related as follows:

$$y[k+1] = y[k](1+\alpha) - x[k], \quad k \geq 0. \qquad y[0] = C.$$

a) Use z-transforms to solve for $y[k]$.

b) How many periods will it take to pay off the loan?

Problem 5. Determine the zero-state response of a system having a transfer function

$$H(z) = \frac{z}{(z+0.2)(z+0.8)} \qquad |z| > 0.8$$

and an input given by

$$x[n] = 2^n u[-(n+1)].$$

Problem 6. A causal LTI system has rational transfer function $H(s)$. When appropriate, assume all initial conditions are zero.

a) Is it possible for this system to output $y(t) = \sin(100\pi t)u(t)$ in response to an input $x(t) = \cos(100\pi t)u(t)$? Explain.

b) Is it possible for this system to output $y(t) = \sin(100\pi t)u(t)$ in response to an input $x(t) = \cos(50\pi t)u(t)$? Explain.

c) Is it possible for this system to output $y(t) = \sin(100\pi t)$ in response to an input $x(t) = \cos(100\pi t)$? Explain.

Problem 7. Establish the following transform pairs. Show all work (i.e., do not quote results from a table).

a) If $x[n]$ has unilateral z-transform $\mathcal{X}(z)$ with ROC $= R$, then

$$\gamma^n x[n]u[n] \longleftrightarrow \mathcal{X}(z/\gamma).$$

Make sure to specify how the ROC changes.

b) If $x_i(t)$ has bilateral Laplace transform $X_i(s)$, then

$$x_1(t) * x_2(t) \longleftrightarrow X_1(s)X_2(s).$$

Make sure to specify how the ROC relates to R_i =ROC for $x_i(t)$, $i = 1, 2$.

Problem 8. For a vector $\mathbf{x} = \{x[0], x[1], \ldots, x[N-1]\}$, its N-point DFT $\mathbf{X} = \{X[0], \ldots, X[N-1]\}$ may be computed via a matrix-vector multiplication

$$\mathbf{X} = D\mathbf{x},$$

where D is an $N \times N$ matrix.

a) Write an explicit expression for the (i, j)-th entry of D (i.e., the entry in the ith row, and jth column).

b) What are the (right-)eigenvectors and associated eigenvalues of the matrix D?

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