EECS 120
Midterm 2
Wed. April 9, 2014
1610-1730 pm
Name: $\qquad$
SID: $\qquad$

- Closed book. One $8.5 \times 11$ inch page two sides formula sheet (or two single sided). No calculators.
- There are 4 problems worth 100 points total. There may be more time efficient methods to solve problems.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 24 |  |
| 2 | 28 |  |
| 3 | 22 |  |
| 4 | 26 |  |
| TOTAL | 100 |  |

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

| $\tan ^{-1} 0.1=5.7^{\circ}$ | $\tan ^{-1} 0.2=11.3^{\circ}$ |
| :---: | :---: |
| $\tan ^{-1} \frac{1}{2}=26.6^{\circ}$ | $\tan ^{-1} 1=45^{\circ}$ |
| $\tan ^{-1} \frac{1}{3}=18.4^{\circ}$ | $\tan ^{-1} \frac{1}{4}=14^{\circ}$ |
| $\tan ^{-1} \sqrt{3}=60^{\circ}$ | $\tan ^{-1} \frac{1}{\sqrt{3}}=30^{\circ}$ |
| $\sin 30^{\circ}=\frac{1}{2}$ | $\cos 60^{\circ}=\frac{\sqrt{3}}{2}$ |
| $\cos 45^{\circ}=\frac{\sqrt{2}}{2}$ | $\sin 45^{\circ}=\frac{\sqrt{2}}{2}$ |


| $20 \log _{10} 1=0 d B$ | $20 \log _{10} 2=6 d B$ |  |
| :---: | :---: | :---: |
| $20 \log _{10} \sqrt{2}=3 d B$ | $20 \log _{10} \frac{1}{2}=-6 d B$ |  |
| $20 \log _{10} 5=20 d b-6 d B=14 d B$ | $20 \log _{10} \sqrt{10}=10 \mathrm{~dB}$ |  |
| $1 / e \approx 0.37$ | $1 / e^{2} \approx 0.14$ | $\sqrt{2} \approx 1.41$ |
| $1 / e^{3} \approx 0.05$ | $\sqrt{10} \approx 3.16$ | $\sqrt{3} \approx 1.73$ |

## Problem 1. Superheterodyne receiver (24 pts)

Two signals $x_{1}(t)$ and $x_{2}(t)$ have spectra (Fourier Transforms) $X_{1}(j \omega)$ and $X_{2}(j \omega)$ respectively, as shown in the figure below. You are given a signal $y(t)$ with spectrum $Y(j \omega)$, which contains 2 received signals, $x_{1}(t) \cos \left(\omega_{1} t\right)$ (from station 1) and $x_{2}(t) \cos \left(\omega_{2} t\right)$ (from station 2). A narrow bandpass filter $H_{I F}(j \omega)$ with fixed center frequency $\omega_{I F}<\omega_{1}$ is used to filter out interference.


The block diagram below, when completed, should recover the original $x_{2}(t)$ from $y(t)$.
[7 pts] a. Add signal processing elements as necessary in block1 such that the output $Z(j \omega)$ will have $X_{2}(j \omega)$ in the passband of the bandpass filter. For each element, specify amplitudes and frequencies as necessary.
[10 pts] b. For your block1 system, sketch the spectra $Z(j \omega)$, labelling key frequencies and amplitudes.
[7 pts] c. Add signal processing elements as necessary in block2 such that the output of block2 is $x_{2}(t)$. For each element, specify amplitudes and frequencies as necessary.

$Z(j \omega)$

| $\mid$ | -1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-2 \omega 2$ | $-\omega 2$ | $-\omega 1$ | $-\omega_{\mathrm{IF}}$ | 0 | $\omega_{\mathrm{IF}}$ | $\omega 1$ | $\omega 2$ |

## Problem 2. Sampling and Discrete Fourier Transform (28 pts)

For parts a) and b), consider the system below, where $x(t)=\cos (6 \pi t)$. Parts a) and b) may have different $w(t), T_{s}, T_{o}$ and should be answered independently. Sketch should label peak magnitude, and frequency of zero crossing(s) should match given scale.
Note $\Pi(t)=u(t+0.5)-u(t-0.5)$.

[7 pts] a. Given $x(t)=\cos (6 \pi t), \quad w(t)=\Pi(3 t)$. Sketch $\operatorname{Re}\left\{X_{w}(j \omega)\right\}$, where $X_{w}(j \omega)=\mathcal{F}\left\{x_{w}(t)\right\}$ :


## Problem 2. cont.

[7 pts] b. Given a windowed and sampled signal $x_{2 \delta}(t)$ with spectrum $\left.X_{2 \delta}(j \omega)\right\}$ :


Sketch $\operatorname{Re}\left\{X_{2}^{\prime}(j \omega)\right\}$ where $X_{2}^{\prime}(j \omega)=\mathcal{F}\left\{x_{2}^{\prime}(t)\right\}$, given $T_{o}=2 / 3 \mathrm{sec}$.


## Problem 2. cont.

[7 pts] c. Given $x_{3}[n]=\cos (\pi n / 4)$, sketch $X_{3}[k]$, the 16 point DFT of $x_{3}[n]$, labelling amplitudes.

[7 pts] d. Given $x_{4}[n]=x_{3}[n]=\cos (\pi n / 4)$ for $n$ even, and $x_{4}[n]=0$ for $n$ odd, sketch $X_{4}[k]$, the 16 point DFT of $x_{4}[n]$, labelling amplitudes.


## Problem 3. Laplace Transform (22 points)

A causal system with input $x(t)$ and output $y(t)$ is described by the differential equation:

$$
\frac{d^{2}}{d t^{2}} y(t)+5 \frac{d}{d t} y(t)+6 y(t)=x(t) .
$$

[11 pts] a. Find $Y(s)$ and $y(t)$ for $x(t)=0(\mathrm{ZIR}) . y\left(0^{-}\right)=1, \quad \frac{d}{d t} y\left(0^{-}\right)=2$.
$\qquad$
$Y(s)=$
$y(t)=$ $\qquad$
[11 pts] b. Find $Y(s)$ and $y(t)$ for $x(t)=u(t)(\mathrm{ZSR}) . y\left(0^{-}\right)=0, \quad \frac{d}{d t} y\left(0^{-}\right)=0$.
$\qquad$
$Y(s)=$

$$
y(t)=
$$

## Problem 4. Feedback System (26 points)


[4 pts] a. Find the transfer function for the system above which has input $x(t)$ and output $y(t)$.

$$
H(s)=\frac{Y(s)}{X(s)}=
$$

[6 pts] b. Find the frequency $\omega_{o}$ at which the phase of $G(s)$ is $-180^{\circ}$.

$$
\omega_{o}=
$$

[6 pts] c. For the frequency $\omega_{o}$ found above, what is the maximum $K$ which could be used before the closed-loop system is unstable (this is the gain margin).

$$
K<
$$

[6 pts] d. Find the sinusoidal steady state response of the closed-loop system with $K=4$ to the input $x(t)=\cos (t) u(t)$, ignoring any transients. (Hint: phasors).

$$
y(t)=
$$

[4 pts] e. Without explicitly calculating, discuss the steady-state closed-loop response of the system to $x(t)=\cos \left(\omega_{o} t\right) u(t)$ using $\omega_{o}$ from part b and the $K$ value from part c.

