EECS 120
Midterm 1
Wed. Feb. 26, 2014
1610 - 1730 pm

Name:____________________________
SID:_____________

- Closed book. One 8.5x11 inch page one side formula sheet. No calculators.

- There are 4 problems worth 100 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	21	
2	27	
3	25	
4	27	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1} 1 = 45°$
$\tan^{-1}\frac{1}{3} = 18.4°$	$\tan^{-1}\frac{1}{4} = 14°$
$\tan^{-1}\sqrt{3} = 60°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\sin 30° = \frac{1}{2}$	$\cos 60° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB
$1/e \approx 0.37$	$1/e^2 \approx 0.14$
$1/e^3 \approx 0.05$	$\sqrt{10} \approx 3.16$

Problem 1 LTI Properties (21 pts)

[15 pts] Classify the following systems, with input $x(t)$ and output $y(t)$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).

System	Causal	Linear	Time-invariant
a. $y(t) = x(t)\cos(2\pi t)$			
b. $y(t) = x(t) * u(t-2)$			
c. $y(t) = 3x(t+1) + 1$			
d. $y(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$			
e. $y(t) = x(t) - \frac{1}{2}\frac{dy(t)}{dt}$			

[6 pts] Two of the systems above (a,b,c,d,e) are not BIBO stable. Note below which systems are not BIBO stable, and then find a bounded input $x(t)$ which gives rise to an unbounded output $y(t)$ for each of these systems.

System 1:

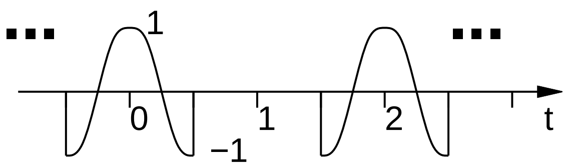
Bounded input $x(t) =$ __________

System 2:

Bounded input $x(t) =$ __________

Problem 2 Fourier Series (27 pts)

You are given a periodic function $x(t)$ as shown, where the shape is one period of a cosine:



$x(t)$ can be represented by a Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jk\omega_o t}$$

.

[2 pts] a. What is the fundamental frequency $\omega_o =$ __________

[8 pts] b. Find $x_k =$ __________________________

Problem 2, continued.

A periodic signal $c(t)$ with period $T_o = 2$ seconds has complex Fourier series coefficients c_k where:

$$c_k = \frac{1}{2}\left[\frac{\sin\frac{(1-k)\pi}{2}}{\pi(1-k)} + \frac{\sin\frac{(1+k)\pi}{2}}{\pi(1+k)}\right]$$

[2 pts] c. What is the time average DC power in $c(t)$? __________

[3 pts] d. What is the time average power in the fundamental frequency component ω_o? __________

The signal $c(t)$ is passed through a filter with frequency response $H(j\omega)$, with:

$$H(j\omega) = 1 - e^{-j\omega},$$

and the output of the filter is $d(t)$, where $d(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\pi t}$.

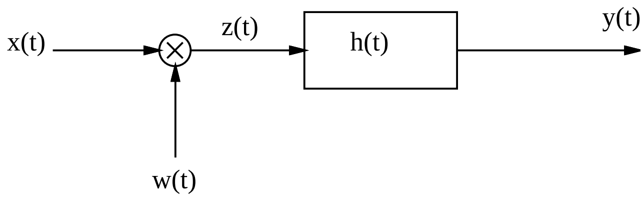
[8 pts] e. Find d_k (leave as an expression) = __________

[4 pts] f. Complete the table for specific frequency components, simplifying if possible:

k	d(k)
0	
1	
2	
3	

Problem 3. Fourier Transform (25 pts)

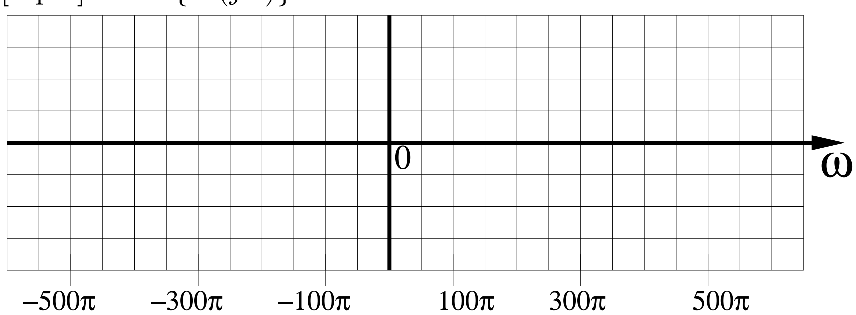
For each part below, consider the following system:



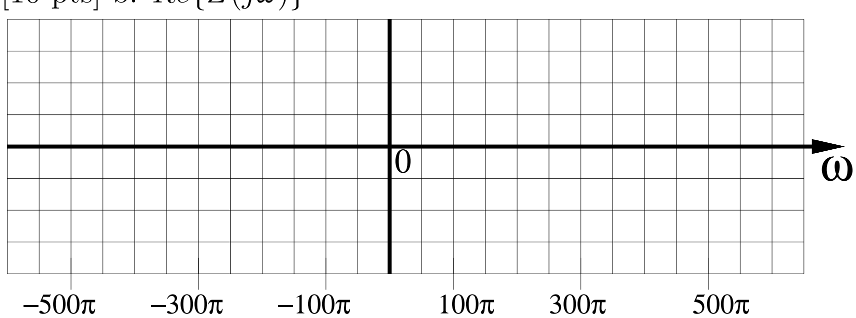
Where $x(t) = \cos(400\pi t) + \frac{\sin 100\pi t}{\pi t}$, $\quad w(t) = \frac{\sin 50\pi t}{\pi t}$, $\quad h(t) = \frac{\sin 100\pi t}{\pi t}$

Sketch $ReX(j\omega), ReZ(j\omega), ReY(j\omega)$ labelling height/area, center frequencies, and key zero crossings for $-500\pi \leq \omega \leq 500\pi$:

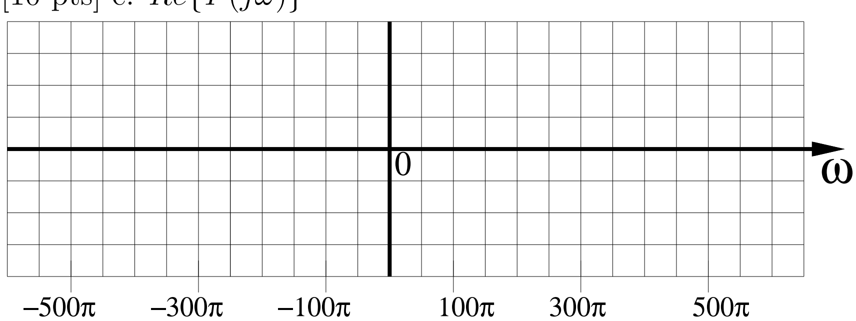
[5 pts] a. $Re\{X(j\omega)\}$



[10 pts] b. $Re\{Z(j\omega)\}$



[10 pts] c. $Re\{Y(j\omega)\}$



Problem 4. DTFT (27 points)

A causal LTI system with input $x[n]$ and output $y[n]$ is described by the difference equation:

$$y[n] - y[n-1] = x[n]$$

[4 pts] a. Find $H(e^{j\omega})$ the transfer function for the system = __________

[2 pts] b. Find the impulse response $h[n]$, that is, the time response of the system to input $x[n] = \delta[n]$.

$h[n] =$__________

[10 pts] c. If $x[n] = 2\cos(\frac{1}{2}\pi n)$ find $y[n]$. $y[n] =$ __________

[4 pts] d. Show that $y[n]$ is real in part c (above).

Problem 4, continued.

[4 pts] e. An LTI system has transfer function $K(e^{j\omega}) = \frac{L(e^{j\omega})}{M(e^{j\omega})} = 2\cos(\omega) + 2j\sin(2\omega)$.

Write the difference equation for this system with input $m[n]$ and output $l[n]$:

$l[n] =$ ______________________

[3 pts] f. A signal $x[n]$ has DTFT $X(e^{j\omega})$.
Another signal $y[n] = x[-2n+4]$. Find the DTFT of $y[n]$ in terms of $X(e^{j\omega})$.

$Y(e^{j\omega}) =$ ______________________