EECS 120
Final Exam
Fri. May 16, 2014
0810 - 1100 am

Name:___________________________
SID:_____________

- Closed book. Three single sided 8.5x11 inch pages of formula sheet. No calculators.
- There are 8 problems worth 200 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	24	
2	29	
3	14	
4	29	
5	24	
6	34	
7	24	
8	22	
TOTAL	200	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

$\tan^{-1} 0.1 = 5.7°$	$\tan^{-1} 0.2 = 11.3°$
$\tan^{-1} \frac{1}{2} = 26.6°$	$\tan^{-1} 1 = 45°$
$\tan^{-1} \frac{1}{3} = 18.4°$	$\tan^{-1} \frac{1}{4} = 14°$
$\tan^{-1} \sqrt{3} = 60°$	$\tan^{-1} \frac{1}{\sqrt{3}} = 30°$
$\sin 30° = \frac{1}{2}$	$\cos 60° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20 \log_{10} 1 = 0dB$	$20 \log_{10} 2 = 6dB$	
$20 \log_{10} \sqrt{2} = 3dB$	$20 \log_{10} \frac{1}{2} = -6dB$	
$20 \log_{10} 5 = 20db - 6dB = 14dB$	$20 \log_{10} \sqrt{10} = 10$ dB	
$1/e \approx 0.37$	$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$
$1/e^3 \approx 0.05$	$\sqrt{10} \approx 3.16$	$\sqrt{3} \approx 1.73$

Problem 1 LTI Properties (24 pts)

[24 pts] Classify the following systems, with input $x(t)$ (or $x[n]$) and output $y(t)$ (or $y[n]$). In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).

System	Causal	Linear	Time-invariant	BIBO stable
a. $y(t) = x(t) * \sum_{n=0}^{\infty} \delta(t-4n)$				
b. $y(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-4n)$				
c. $y[n] = x[n] * 0.9^n u[n]$				
d. $y[n] = \sum_{m=0}^{\infty} x[m] 0.9^m$				
d. $y(t) = x(t) * [\delta(t+1) + e^{-2t}u(t)]$				
e. $y(t) = x(t) * [\frac{d}{dt}\delta(t-1) + e^{-2t}u(t)]$				

Problem 2 Short Answers (29 pts)

Answer each part independently. Note $\Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$.

[3 pts] a. Evaluate $\delta(t + \frac{1}{4}) * \cos(2\pi t)u(t) =$ _________

[4 pts] b. Sketch $\Pi(t-1) * \Pi(2t)$

[4 pts] c. Given an LTI system with input $x(t) = \Pi(t - 4.5)$ and output $y(t) = \Pi(2t) - \Pi(2(t-1))$, find the impulse response of the system,

$h(t) =$ _________. (Hint: sketch $x(t)$ and $(y(t))$)

[4 pts] d. Given $x(t) = \sum_{n=-\infty}^{\infty}[\delta(t-2n) + \frac{1}{2}\delta(t-2n+1)]$, find $X(j\omega)$ the Fourier Transform of $x(t)$.

$X(j\omega) =$_________

[4 pts] e. What is the energy in the time signal $\frac{\sin(100\pi t)}{t}$? __________

[6 pts] f. A system with input $x(t)$ and output $y(t)$ is described by the following differential equation: $\frac{d^2}{dt^2}y + 2\frac{d}{dt}y + y = \frac{d}{dt}x + 2x$.

Assuming zero initial conditions, find the impulse response for this system.

$h(t) =$ __________

[4 pts] g. Given the bilateral Laplace transform $X(s) = \frac{1}{s-2}$ and the region of convergence is to the left of the pole at $s = 2$, find the inverse Laplace transform,

$x(t) =$ __________

Problem 3 Laplace Transform (14 pts)

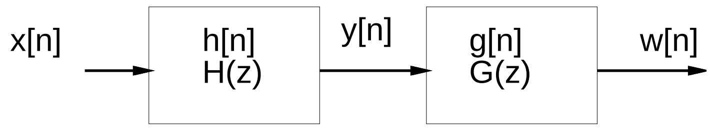
[14 pts] An LTI system has input $x(t)$, output $y(t)$ and transfer function

$$H(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For input $x(t) = (1 + \cos\omega_n t)u(t)$, find the steady-state solution $y(t)$ for large t.

$y(t) =$______________

Problem 4. Z transform (29 pts)



[10 pts] a. Consider an LTI causal system with impulse response $h[n] = (2 - (\frac{1}{2})^n)u[n]$. Find $g[n]$ such that $h[n] * g[n] = \delta[n]$.

$g[n] =$____________

[3 pts] b. Show by direct calculation that the $g[n]$ from part a. above is the inverse of $h[n]$ from part a.

[4 pts] c. Show that $g[n]$ is the impulse response of a stable system.

[12pts] d. Consider an LTI causal system with Z transform

$$H(z) = \frac{z(z-2)}{z - 3/4}$$

.

Find a **stable** $G(z)$ such that $|H(e^{j\Omega})G(e^{j\Omega})| = 1$ for all Ω.

$G(z) =$____________

Problem 5. Z Transform (24 pts)

A causal system with input $x[n]$ and output $y[n]$ is described by the difference equation:

$$y[n] + 0.3y[n-1] - 0.4y[n-2] = x[n] - x[n-1]$$

[12 pts] a. Find $Y(z)$ and $y[n]$ for $x[n] = 0$ (ZIR), with $y[-2] = 4$ and $y[-1] = 2$.

$Y(z) =$ ____________________ $y[n] =$ ____________________

[12 pts] b. Find $Y(z)$ and $y[n]$ for $x[n] = u[n]$ (ZSR). $y[-2] = 0$ and $y[-1] = 0$.

$Y(z) =$ ____________________ $y[n] =$ ____________________

Problem 6. Digital Filter (34 pts)

A continuous time causal LTI filter has transfer function

$$H(s) = 4\frac{s+1}{s+4}$$

[4 pts] a. Find the linear differential equation with constant coefficients with input $x(t)$ and output $y(t)$ which has transfer function $H(s)$ (assume zero initial conditions).

LDE:________________

[4 pts] b. Using the backward difference approximation for the derivative (i.e.

$\frac{dy}{dt} \approx \frac{y[n]-y[n-1]}{T}$),

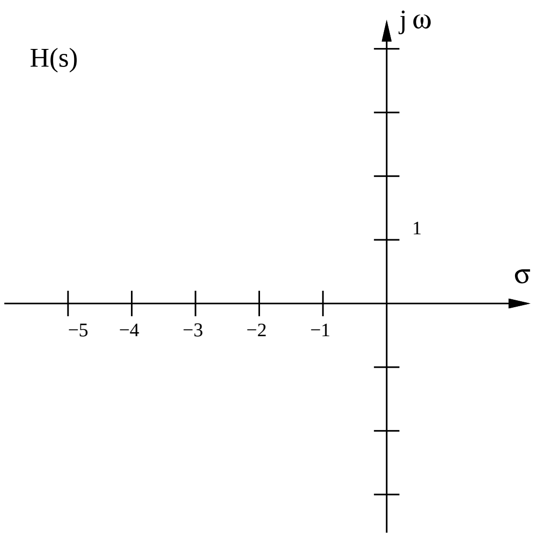
with $T = \frac{1}{2}$, find the linear difference equation approximation with input $x[n]$ and output $y[n]$.

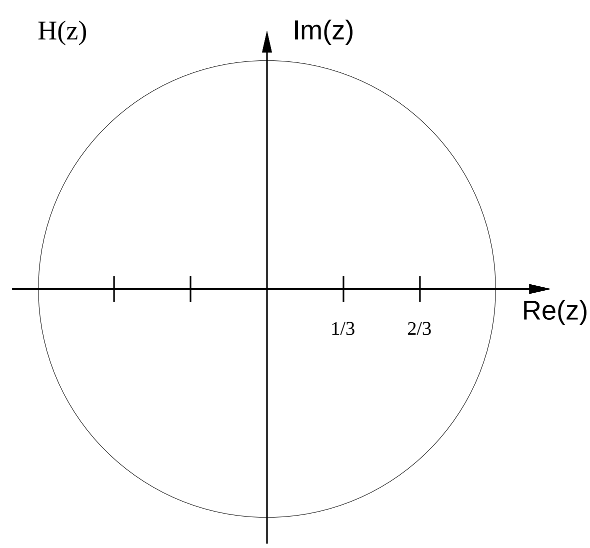
LDE:________________

[4 pts] c. Assuming zero initial conditions, find the Z transform for the LDE in part b.

$H(z) =$ ________________

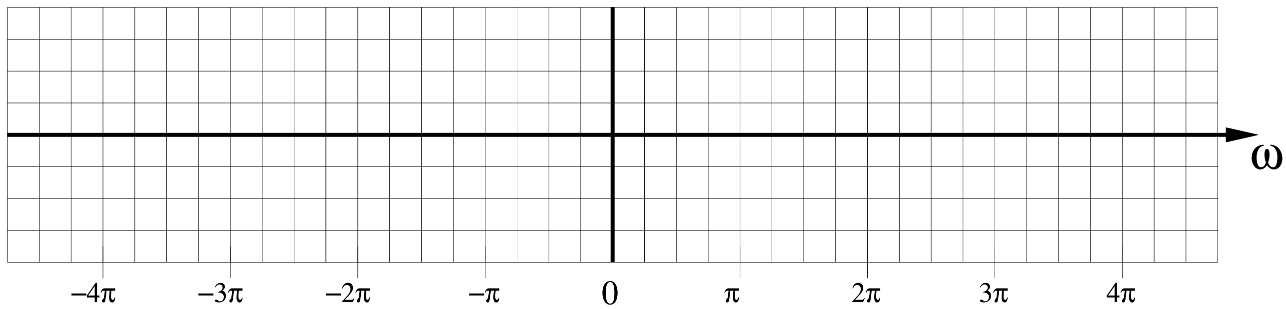
[4 pts] d. Draw pole-zero diagrams for $H(s)$ in the s-plane and $H(z)$ in the z-plane.





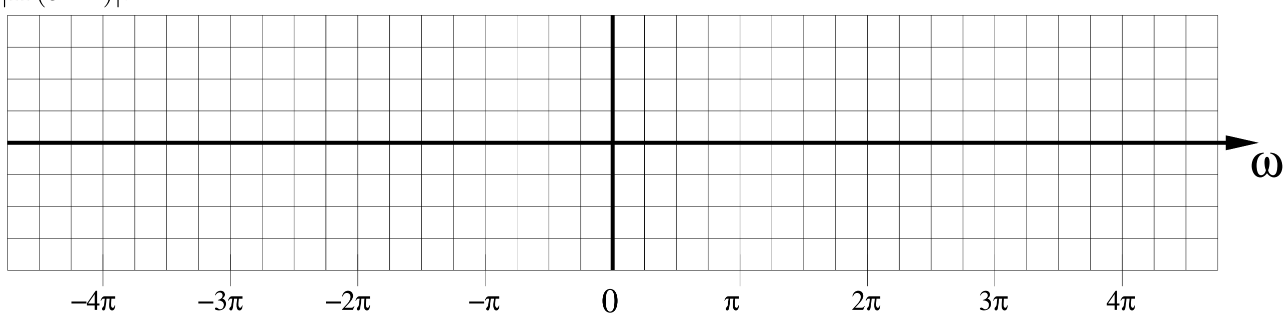
[8 pts] e. Sketch the magnitude of frequency response of the continuous time system, labelling key amplitudes.

$|H(j\omega)|$:



[8 pts] f. Sketch the magnitude of frequency response of the discrete time system, noting that $T = \frac{1}{2}$ sec., labelling key amplitudes.

$|H(e^{j\omega T})|$:

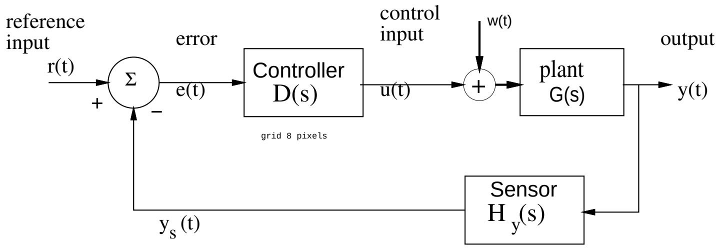


[2 pts] g. Briefly explain the reasons for differences between the magnitudes of the CT and DT frequency responses.

Some possibly useful constants:

$\pi \approx 3.14$	$2\pi \approx 6.3$
$3\pi \approx 9.42$	$4\pi \approx 12.6$
$\sqrt{2} \approx 1.4$	$\sqrt{3} \approx 1.7$
$\sqrt{5} \approx 2.2$	$\sqrt{10} \approx 3.2$
$\sqrt{8} \approx 2.8$	$\sqrt{17} \approx 4.1$
$\sqrt{20} \approx 4.5$	$\sqrt{26} \approx 5.1$

Problem 7. Control (24 pts)



[3 pts] a. Find the transfer function $\frac{E(s)}{R(s)}$ in terms of D, G, H_y.

$\frac{E(s)}{R(s)} =$ ________________

[3 pts] b. Find the transfer function $\frac{E(s)}{W(s)}$ in terms of D, G, H_y.

$\frac{E(s)}{W(s)} =$ ________________

For the system above, let $D(s) = k_p$, $H_y(s) = \frac{s+1}{s}$, and $G(s) = \frac{1}{s^2+as+b}$.

[10 pts] c. With $r(t) = 0$, determine trend of $e(t)$ as $t \to \infty$ with respect to a step disturbance input $w(t)$.

$e(t) \to$ ________________

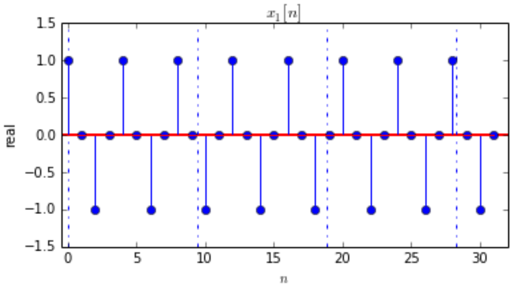
[8 pts] d. With $w(t) = 0, H_y(s) = 1, D(s)G(s) = \frac{200}{(s+1)^2(s+10)^2}$,
determine ω_c for which $|D(j\omega_c)G(j\omega_c)| \approx 1$ and the approximate phase margin.
(Hint for small angles $\tan^{-1} 0.1 \approx 0.1$ rad $\approx 5.7°$.)

$\omega_c =$ ________________

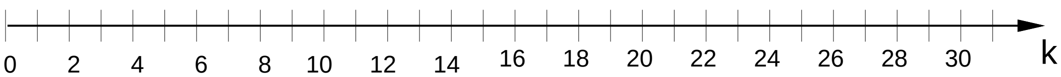
phase margin (specify rad or degrees) ________________

Problem 8. DFT problem or pole-zero match (22 pts)

[11 pts] a. Given $x_1[n] = \cos(2\pi \frac{n}{4})$ as shown:

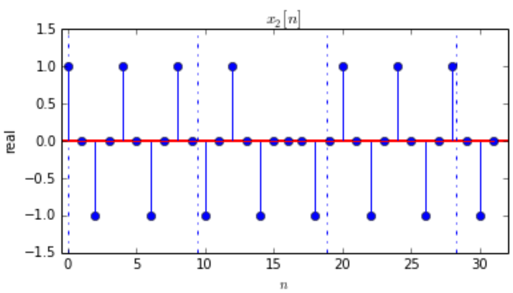


sketch $X_1[k]$, the 32 point DFT of $x_1[n]$, labelling amplitudes.

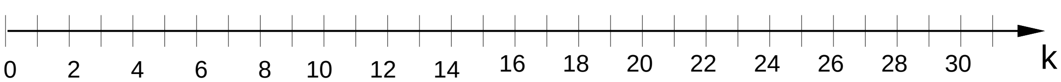


$X_1[k]$:

[11 pts] b. Given $x_2[n] = \cos(2\pi \frac{n}{4}) - \delta[n-16]$ as shown:



sketch $X_2[k]$, the 32 point DFT of $x_2[n]$, labelling amplitudes.



$X_2[k]$:

Area for scratch work.