EE 120: Signals and Systems MIDTERM 1
Department of EECS 24 February 2010
UC BERKELEY

LAST Name ______________ FIRST Name ______________

Discussion Time ______________

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (35 Points) The frequency response of a causal discrete-time LTI filter H is

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = A\, \frac{1 - e^{-i2\omega}}{1 + R^2\, e^{-i2\omega}},$$

where $R^2 = 0.96$ and $A = 1/50$.

(a) Determine the linear, constant-coefficient difference equation that characterizes the input-output behavior of the system.

(b) Provide a well-labeled plot of $|H(\omega)|$, the magnitude response of the filter. You must explain how you arrive at the plot.

(c) Determine the output of the filter in response to the input signal

$$\forall n \in \mathbb{Z}, \quad x(n) = 1 + (-1)^n + \cos\left(\frac{\pi}{2} n\right).$$

MT1.2 (35 Points) A continuous-time signal x is periodic with fundamental period $p = 6$ seconds. We sample this signal every $T = 3$ seconds to produce a discrete-time signal g as follows:

$$\forall n \in \mathbb{Z}, \quad g(n) = x(nT).$$

(a) Show that $g(n+2) = g(n)$, for all n.

(b) Express the DFS coefficients G_ℓ of the DT signal g in terms of the CTFS coefficients X_k of the CT signal x.

(c) Explain whether g is guaranteed to be periodic if p and T are arbitrary positive real numbers.

MT1.3 (35 Points) The spectrum of a periodic DT signal x is given by

$$\forall \omega \in \mathbb{R}, \quad X(\omega) = \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{\pi}{3} + 2\pi k\right) + \delta\left(\omega - \frac{3\pi}{5} + 2\pi k\right),$$

(a) Determine a reasonably simple expression for $x(n)$, for all n.

(b) Determine the fundamental period, the fundamental frequency, and the DFS coefficients of x. How many of the coefficients are zero?

Basic Formulas:

Discrete Fourier Series (DFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \ ,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\displaystyle\sum_{k=\langle p \rangle}$ may denote $\displaystyle\sum_{k=0}^{p-1}$ or $\displaystyle\sum_{k=1}^{p}$.

Continuous-Time Fourier Series (FS) Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period p:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ik\omega_0 t} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt \ ,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable continuous interval of length p. For example, $\displaystyle\int_{\langle p \rangle}$ can denote $\displaystyle\int_0^p$.

Discrete-Time Impulse Response and Frequency Response If h and H denote the impulse response and frequency response, respectively, of a DT-LTI system, then

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) \, e^{-i\omega n}, \quad \forall \omega \in \mathbb{R} \, ,$$

and

$$h(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} H(\omega) \, e^{i\omega n} \, d\omega \, .$$

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Problem	Points	Your Score
Name	10	
1	35	
2	35	
3	35	
Total	**115**	