EE 120: Signals and Systems
Department of EECS
UC BERKELEY

MIDTERM 1
18 February 2009

LAST Name Philter FIRST Name Beebo

Discussion Time It's so clear, I don't need discussion.

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

Derivation : MT 1.4.

$$f(n) \longrightarrow \boxed{\uparrow N} \longrightarrow g(n)$$

$$G_k = \frac{1}{p_g} \sum_{n=0}^{p_g - 1} g(n) e^{-ik \frac{2\pi}{p_g} n} \qquad p_g = N p_f$$

$$\omega_{0f} = \frac{2\pi}{p_f}$$

$$\Rightarrow G_k = \frac{1}{Np_f} \sum_{n=0}^{Np_f - 1} g(n) e^{-ik \frac{2\pi}{p_f N} n}$$

$$= \frac{1}{N p_f}\left[g(0) + g(N) e^{-ik\omega_{0f}} + g(2N) e^{-ik2\omega_{0f}} + \cdots + g((p_f-1)N) e^{-ik(p_f-1)\omega_{0f}} \right]$$

$$= \frac{1}{N} \underbrace{\left\{ \frac{1}{p_f} \left[f(0) + f(1) e^{-ik\omega_{0f}} + f(2) e^{-ik2\omega_{0f}} + \cdots + f(p_f - 1) e^{-ik(p_f-1)\omega_{0f}} \right] \right\}}_{F_k}$$

$$G_k = \frac{1}{N} F_k \qquad k = 0, 1, \ldots, \underbrace{Np_f - 1}_{p_g - 1}$$

But Note: $F_k = F_{k+p_f}$ (just as $G_k = G_{k+p_g}$)

So $\boxed{G_k = \frac{1}{N} F_{k \bmod p_f}}$

According to this derivation, $g(n) = \cancel{G_0}^{\,0} + G_1 e^{i\frac{\pi}{3}n} + \cancel{G_2 e^{i\frac{2\pi}{3}n}}^{\,0} + G_3 e^{i\pi n} + \cancel{G_4 e^{i\frac{4\pi}{3}n}}^{\,0} + G_5 \underbrace{e^{i\frac{5\pi}{3}n}}_{=e^{-i\frac{\pi}{3}n}}$

$0 = \frac{1}{3} F_0 = G_0 = G_2 = G_4$

$\frac{1}{3} = \frac{1}{3} F_1 = G_1 = G_3 = G_5$

$$g(n) = \frac{1}{3}\left[\left(e^{i\frac{\pi}{3}n} + e^{-i\frac{\pi}{3}n} \right) + (-1)^n \right]$$

$$g(n) = \frac{1}{3}\left[(-1)^n + 2\cos\left(\frac{\pi}{3}n\right) \right]$$

MT1.1 (20 Points) Consider a discrete-time system H shown below:

$$x \longrightarrow \boxed{H} \longrightarrow y$$

If the input to the system is $x(n) = \exp\left(i\frac{3\pi}{4}n\right)$ for all n, the corresponding output is $y(n) = \exp\left(-i\frac{3\pi}{4}n\right)$ for all n.

Can the system be LTI? Explain your reasoning succinctly, but clearly and convincingly.

H can NOT be LTI, because a new frequency appears at the output. We know that LTI systems can not create new frequencies.

If you believe that the system H can be LTI, then determine the impulse response *and* frequency response of an LTI system consistent with the input-output behavior described for H.

If, on the other hand, you believe that the system H cannot be LTI, then

(i) Find a *nonlinear time-invariant* system consistent with the input-output behavior described for H.

$$x \longrightarrow \boxed{(\cdot)^{-1}} \longrightarrow y$$

$y(n) = \dfrac{1}{x(n)}$ will do the trick.

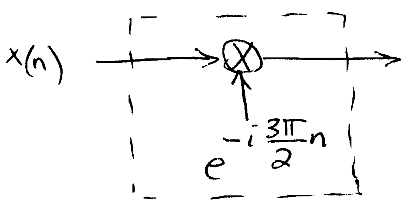
The system $x \longrightarrow \boxed{(\cdot)^*} \longrightarrow y = x^*$ also works.

Q. Why is this system not linear?

Our input signal is never zero, so we don't worry about $\frac{1}{x(n)}$ blowing up.

(ii) Find a *linear time-varying* system consistent with the input-output behavior described for H.

A modulator can shift the frequency $\frac{3\pi}{4}$ to $-\frac{3\pi}{4}$.



We know that a modulator is linear, but time-varying.

MT1.2 (25 Points) The input-output behavior of a discrete-time causal LTI system H is described by the linear constant-coefficient difference equation shown below:

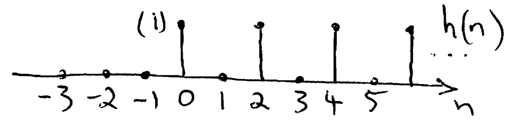
$$y(n) - y(n-2) = x(n).$$

Note that the system must be at initial rest if it is to be LTI.

(a) Determine, and provide a well-labeled stem plot of, the impulse response h.

Let $x(n) = \delta(n) \Rightarrow h(n) = h(n-2) + \delta(n)$ Initial rest means $h(-1) = h(-2) = \cdots = 0$

$$\left.\begin{aligned} h(0) &= h(-2) + \delta(0) = 1 \\ h(1) &= h(-1) = 0 \\ h(2) &= h(0) = 1 \\ h(3) &= h(1) = 0 \\ h(4) &= h(2) = 1 \end{aligned}\right\} \Rightarrow$$



← An upsampled unit step.

$h(n) = \dfrac{1+(-1)^n}{2} u(n)$ ← not required for full credit

(b) Can the system H be BIBO stable?

If you believe that the system *can* be BIBO stable, then determine the frequency response $H(\omega)$, and provide a well-labeled plot of the magnitude response $|H(\omega)|$.

If, on the other hand, you believe that the system *cannot* be BIBO stable, then specify a *bounded* input signal x such that at least one sample $y(n)$ of the corresponding output signal is unbounded.

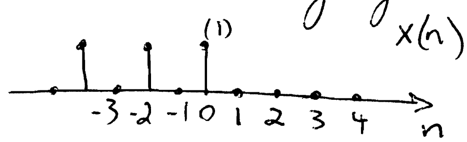
This LTI system is NOT BIBO stable because the impulse response is not absolutely summable; that is,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} \frac{1+(-1)^n}{2} = \infty$$

Absolute summability of the impulse response is a necessary and sufficient condition for BIBO stability.

There must exist a signal that is bounded, in response to which the output is unbounded for at least some n. The following signal will do the trick.

$$x(n) = \begin{cases} \dfrac{h(-n)}{|h(-n)|} & \text{if } h(-n) \neq 0 \\ 0 & \text{if } h(-n) = 0 \end{cases} \rightarrow$$



$x(n)$ turns out to be equal to $h(-n)$.

For this choice of $x(n)$, we can use the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$ and note that $y(0) = \sum_{k=-\infty}^{\infty} h(k) x(-k) = \sum_{k=0}^{\infty} h^2(k) = \infty$

MT1.3 (30 Points) Consider a discrete-time LTI filter H depicted in the figure below.

$$x \longrightarrow \boxed{\;H\;} \longrightarrow y$$

The input-output behavior of the filter is described by the following equation:

$$y(n) = \frac{x(n-1) + x(n) + x(n+1)}{3}.$$

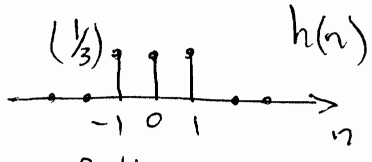
Throughout this problem, explain your reasoning succinctly, but clearly and convincingly.

(a) Can the system H be causal? No, because $y(n)$ depends on the future value $x(n+1)$ of the input.

(b) Determine, and provide a well-labeled plot of, the impulse response h.

Let $x(n) = \delta(n)$. Then the impulse response is simply

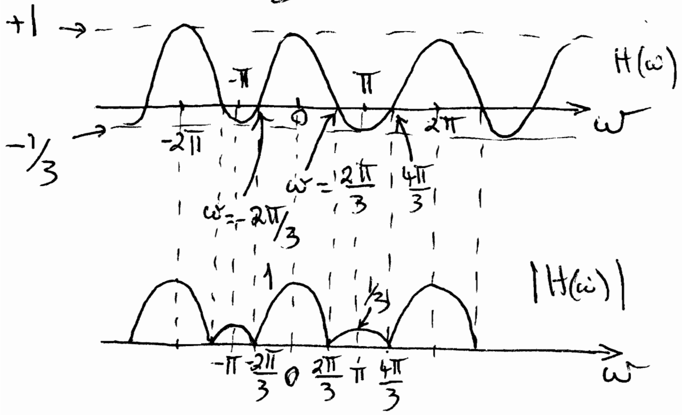
$$h(n) = \frac{\delta(n-1) + \delta(n) + \delta(n+1)}{3}$$



This is a three-point moving-average filter — an FIR filter.

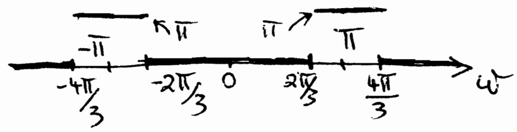
(c) Determine the frequency response values $H(\omega)$ and provide well-labeled plots of the magnitude response values $|H(\omega)|$ and phase response values $\angle H(\omega)$.

$$H(\omega) = \frac{1}{3}\left(e^{i\omega} + 1 + e^{-i\omega}\right) = \frac{1}{3} + \frac{2}{3}\cos\omega = \frac{1}{3}(1 + 2\cos\omega)$$



Note: $\cos\frac{2\pi}{3} = -\frac{1}{2}$

$$\measuredangle H(\omega) = \begin{cases} 0 & \text{if } H(\omega) > 0 \\ \text{undefined} & \text{if } H(\omega) = 0 \\ \pi \text{ or } -\pi & \text{if } H(\omega) < 0 \end{cases}$$



(d) Determine the response of the filter to the input signal

$$x(n) = \cos\left(\frac{7\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right) + (-1)^n + 1.$$

$$\cos\left(\frac{7\pi}{3}n\right) = \cos\left(\frac{7\pi}{3}n - \underbrace{\frac{6\pi}{3}n}_{2\pi n}\right) = \cos\left(\frac{\pi}{3}n\right)$$

$$\cos\left(\frac{\pi}{3}n\right) \longrightarrow \boxed{H} \longrightarrow \underbrace{\left|H\left(\frac{\pi}{3}\right)\right|}_{\frac{1}{3}\left(1+2\cos\frac{\pi}{3}\right) = \frac{2}{3}} \cos\left(\frac{\pi}{3}n + \underbrace{\measuredangle H\left(\frac{\pi}{3}\right)}_{0}\right) = \frac{2}{3}\cos\left(\frac{\pi}{3}n\right)$$

$$\sin\left(\frac{2\pi}{3}n\right) = \frac{e^{i\frac{2\pi}{3}n} + e^{-i\frac{2\pi}{3}n}}{2i} \longrightarrow \boxed{H} \longrightarrow 0$$

because the filter eliminates the frequencies $\pm\frac{2\pi}{3}$ (i.e., $H\left(\frac{2\pi}{3}\right) = H\left(\frac{-2\pi}{3}\right) = 0$)

$$(-1)^n = e^{i\pi n} \longrightarrow \boxed{H} \longrightarrow \underbrace{\underbrace{|H(\pi)|}_{1/3}\, e^{i\pi n}\, e^{i\underbrace{\measuredangle H(\pi)}_{\pi}}}_{\frac{1}{3}e^{i\pi(n+1)} = \frac{-1}{3}e^{i\pi n}}$$

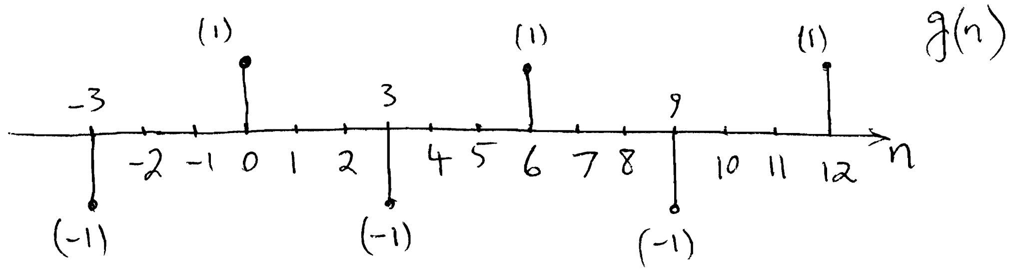
$$1 = e^{i0n} \longrightarrow \boxed{H} \longrightarrow H(0)e^{i0n} = H(0) = 1$$

$$y(n) = \frac{2}{3}\cos\left(\frac{\pi}{3}n\right) + 0 - \frac{1}{3}e^{i\pi n} + 1 = \frac{2}{3}\cos\left(\frac{\pi}{3}n\right) - \frac{1}{3}(-1)^n + 1$$

MT1.4 (30 Points) Consider a periodic discrete-time signal g described as follows:

$$\forall n, \quad g(n) = \begin{cases} +1 & n = 0, \pm 6, \pm 12, \ldots \\ -1 & n = \pm 3, \pm 9, \ldots \\ 0 & \text{elsewhere.} \end{cases}$$

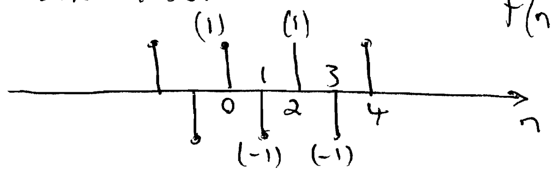
Determine the discrete-time Fourier series coefficients G_k for this signal.



$p_g = 6$

$\omega_{0g} = \frac{2\pi}{6} = \frac{\pi}{3}$

Note that g is simply the upsampled version (by a factor of 3) of the signal f shown below



$f(n) = (-1)^n = e^{i\pi n}$

$$f(n) \longrightarrow \boxed{\uparrow N} \longrightarrow g(n) \qquad \uparrow 3$$

The signal f is periodic with period $p_f = 2$, and fundamental frequency $\omega_{0f} = \pi$.

The DFS coefficients of f are $F_0 = 0$, $F_1 = 1$, $\underbrace{F_2}_{= F_0} = 0$, $\underbrace{F_3}_{= F_1} = 1$

$$G_k = \frac{1}{p_g} \sum_{n=0}^{p_g - 1} g(n) e^{-ik\omega_{0g} n} = \frac{1}{6} \sum_{n=0}^{5} g(n) e^{-ik\frac{2\pi}{6}n} = \frac{1}{6} \sum_{n=0}^{5} g(n) e^{-ik\frac{2\pi}{3}n}$$

You can either evaluate this sum or recall that the coefficients of g and f are related as follows

$$G_k = \frac{1}{N} F_{k \bmod p_f} = \frac{1}{3} F_{k \bmod 2} \qquad k = 0, \ldots, \underbrace{Np_f - 1}_{p_g - 1}$$

$G_0 = \frac{1}{3} F_0 = 0$

$G_1 = \frac{1}{3} F_1 = \frac{1}{3}$

$G_2 = \frac{1}{3} F_{2 \bmod 2} = \frac{1}{3} F_0 = 0$

$G_3 = \frac{1}{3} F_1 = \frac{1}{3}$

$G_4 = \frac{1}{3} F_0 = 0$

$G_5 = \frac{1}{3} F_1 = \frac{1}{3}$

See p. 2 for a full derivation.

LAST Name Philter FIRST Name Beebo

Discussion Time It's so clear, I don't need discussion

Problem Name	Points 10	Your Score 10
1	20	20
2	25	25
3	30	30
4	30	30
Total	**115**	115