EE 120: Signals and Systems
Department of EECS
UC BERKELEY

MIDTERM 1
21 March 2007

LAST Name Philter FIRST Name Anti-Notch
Discussion Time Top Secret

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (10 Points) The following discrete-time systems F and G should be treated mutually independently; properties that hold for one system cannot be *assumed* to hold for the other.

For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) A discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \left(\frac{1}{2}\right)^n u(n), \quad \forall n,$$

in response to the input signal x characterized by $x(n) = \left(\frac{1}{2}\right)^n, \quad \forall n.$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

The input is an exponential, but the output is a truncated exponential, not a scaled version of the input.

(b) A discrete-time system $\mathsf{G} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \left(\frac{1}{4}\right)^n, \quad \forall n,$$

in response to the input signal x characterized by $x(n) = \left(\frac{1}{2}\right)^n, \quad \forall n.$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

The output cannot be written as a scaled version of the input exponential.

MT1.2 (45 Points) Consider an FIR filter $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having impulse response f, frequency response F, and transfer function $\hat{F}$.

$$x \longrightarrow \boxed{\mathsf{F}} \longrightarrow y$$

Each part below discloses partial information about the filter F. Ultimately, your task is to determine the impulse response f completely.

In the space provided for each part, state every inference that you can draw from the information disclosed up to, and including, that part.

Justify all your work succinctly, but clearly and convincingly.

(a) The impulse response f is real-valued, i.e., $f(n) \in \mathbb{R}, \forall n$.

$f(n) \in \mathbb{R} \; \forall n \implies F^*(\omega) = F(-\omega)$ Frequency response is conjugate symmetric.

(b) The frequency response F is real-valued, i.e., $F(\omega) \in \mathbb{R}, \forall \omega$.

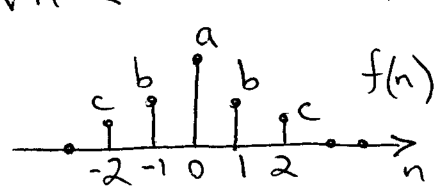
$F(\omega) \in \mathbb{R}, \forall \omega \implies f^*(n) = f(-n)$

But from part (a) we know f is real-valued $\Big\} \implies$

$f(n) = f(-n) \in \mathbb{R}, \forall n$ $\quad$ f is real and even

(c) $\hat{F}(z)$ has *exactly* two poles at $|z| = \infty$.

$f(n) = 0 \quad \forall n < -2 \implies f$ is length-5 and symmetric.



(d) It is known that

$$\sum_{n=\text{ Even Integers}} f(n) = 4.$$

$a + 2c = 4$ (see diagram in part (c))

(e) If the input signal x, characterized by $x(n) = 1, \forall n$, is applied to the filter, the output signal y is $y(n) = 1, \forall n$.

$x(n) = e^{i0n}$, $y(n) = F(0)e^{i0n}$

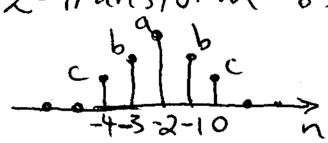
$F(0)$ must be 1 (DC gain is unity)

But $F(0) = \sum_n f(n) \Rightarrow a + 2b + 2c = 1$
From part(d): $a + 2c = 4$ $\Big\} \Rightarrow \underline{b = -3/2}$

(f) It is known that

$$\lim_{z\to 0} z^2 \hat{F}(z) = 1.$$

$z^2 \hat{F}(z)$ is the Z-transform of the function g, where $g(n) = f(n+2), \forall n$.

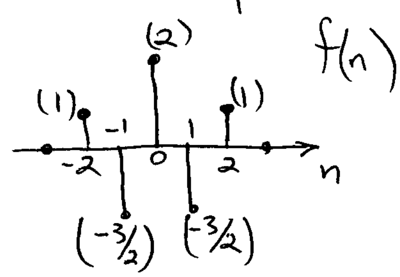


$\lim_{z \to 0} G(z) = g(0) = c \Rightarrow \underline{c = 1}$

Determine, and provide a well-labeled plot of, the impulse response f.

$\left.\begin{array}{l} a + 2c = 4 \\ c = 1 \end{array}\right\} \Rightarrow a = 2$

We have all the information we need to plot f:



MT1.3 (50 Points) Consider a *BIBO stable* discrete-time LTI filter $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:

$$x \longrightarrow \boxed{\mathsf{F}} \longrightarrow y$$

The filter has impulse response f, frequency response F, and rational transfer function $\widehat{F}$.

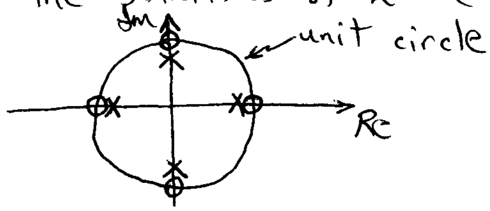
In particular, the transfer function is given by

$$\widehat{F}(z) = \frac{z^4 - 1}{z^4 - (0.99)^4}.$$

(a) Provide a well-labeled pole-zero diagram for $\widehat{F}$.

Zeroes are the solutions of $z^4 = 1 \Rightarrow 0, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}$

Poles are the solutions of $z^4 = (0.99)^4 \Rightarrow 0.99, 0.99e^{i\pi/2}, 0.99e^{i\pi},$ and $0.99e^{i3\pi/2}$



(b) Select the strongest true assertion from the list below. Explain your selection succinctly, but clearly and convincingly.

$\boxed{\text{(i) The system must be } \textit{causal.}}$

(ii) The system could be *causal*, but does not have to be.

(iii) The system cannot be *causal*.

We're told that the system is stable. $\Longrightarrow$

RoC_f must include the unit circle $\Longrightarrow$

$RoC_f = \{z \mid |z| > 0.99\}$

This corresponds to a causal system.

(c) Determine the linear, constant-coefficient difference equation governing the input-output behavior of the filter F.

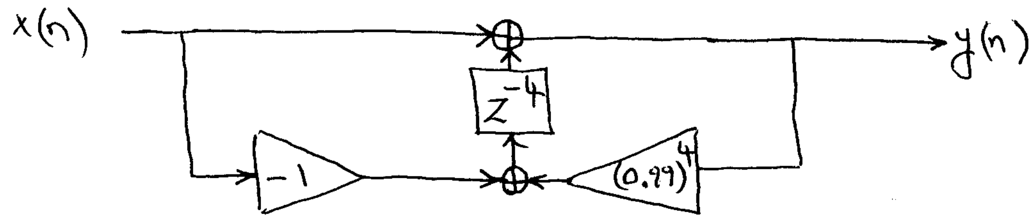
$$\hat{F}(z) = \frac{1 - z^{-4}}{1 - (0.99)^4 z^{-4}} \Longrightarrow y(n) - (0.99)^4 y(n-4) = x(n) - x(n-4)$$

Causal Implementation:

$$y(n) = (0.99)^4 y(n-4) + x(n) - x(n-4)$$

(d) Provide a well-labeled delay-adder-gain block diagram implementation of the filter. Your implementation must employ the minimal number of delay elements possible.

You may coalesce multiple delays into one block, e.g., use a single z^{-N} delay block instead of drawing N one-sample delay blocks in cascade.



The minimal number of delay elements is <u>four</u>.

(e) For the frequency range $-\pi < \omega \leq +\pi$, provide a well-labeled sketch of $|F(\omega)|$, the magnitude response of the filter F.

Is the filter F low-pass, band-pass, high-pass, notch, anti-notch[1], or none of these?

For all points not in the vicinity of $\omega = 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$ all the pole-zero vector pairs have a neutral effect on the magnitude and phase response. The zeroes on the unit circle at those frequencies notch out those frequencies in the input signal.

(f) Determine the response y of the filter F, if the input signal x is

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{m=-\infty}^{\infty} \delta(n - 4m).$$

x is periodic with period $p = 4$. Hence, it has a DFS expansion: $x(n) = \sum_{k=0}^{3} X_k e^{ik\frac{2\pi}{4}n} = \sum_{k=0}^{3} X_k e^{ik\frac{\pi}{2}n}$

All these frequencies, $\omega_0 = \frac{\pi}{2}, 2\omega_0 = \pi, 3\omega_0 = \frac{3\pi}{2}$, and $0\omega_0 = 0$, get notched out.

$y(n) = 0$

(g) Consider a DT-LTI filter G whose impulse response is

$$\forall n \in \mathbb{Z}, \quad g(n) = \left[(0.99)^4\right]^n u(n) - \left[(0.99)^4\right]^{n-1} u(n-1).$$

Determine an explicit relationship between the impulse responses f and g.

$$\hat{G}(z) = \frac{1}{1-(0.99)^4 z^{-1}} - \frac{z^{-1}}{1-(0.99)^4 z^{-1}} = \frac{1 - z^{-1}}{1-(0.99)^4 z^{-1}}$$

We note that $\hat{F}(z) = \hat{G}(z^4) \implies f$ must be a quadruple-upsampled version of g, i.e.,

$$f(n) = \begin{cases} g\left(\frac{n}{4}\right) & n \bmod 4 = 0 \\ 0 & e/w \end{cases}$$

[1]An anti-notch filter notches up, instead of down, at one or more frequencies, and is flat elsewhere.

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~~Lab~~ Discussion Time Top Secret

Problem Name	Points 10	Your Score 10
1	10	10
2	45	45
3	50	50
Total	**115**	115