EE 120: Signals and Systems MIDTERM 1
Department of EECS 21 March 2007
UC BERKELEY

LAST Name ______________________ FIRST Name ______________________

Discussion Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (10 Points) The following discrete-time systems F and G should be treated mutually independently; properties that hold for one system cannot be *assumed* to hold for the other.

For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) A discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \left(\frac{1}{2}\right)^n u(n), \quad \forall n,$$

in response to the input signal x characterized by $x(n) = \left(\frac{1}{2}\right)^n, \quad \forall n.$

Select the strongest true assertion from the list below.

(i) The system must be LTI.
(ii) The system could be LTI, but does not have to be.
(iii) The system cannot be LTI.

(b) A discrete-time system $\mathsf{G} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

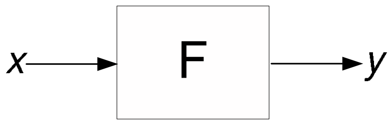
$$y(n) = \left(\frac{1}{4}\right)^n, \quad \forall n,$$

in response to the input signal x characterized by $x(n) = \left(\frac{1}{2}\right)^n, \quad \forall n.$

Select the strongest true assertion from the list below.

(i) The system must be LTI.
(ii) The system could be LTI, but does not have to be.
(iii) The system cannot be LTI.

MT1.2 (45 Points) Consider an FIR filter $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having impulse response f, frequency response F, and transfer function $\widehat{F}$.



Each part below discloses partial information about the filter F. Ultimately, your task is to determine the impulse response f completely.

In the space provided for each part, state every inference that you can draw from the information disclosed up to, and including, that part.

Justify all your work succinctly, but clearly and convincingly.

(a) The impulse response f is real-valued, i.e., $f(n) \in \mathbb{R}, \forall n$.

(b) The frequency response F is real-valued, i.e., $F(\omega) \in \mathbb{R}, \forall \omega$.

(c) $\widehat{F}(z)$ has *exactly* two poles at $|z| = \infty$.

(d) It is known that

$$\sum_{n=\text{ Even Integers}} f(n) = 4.$$

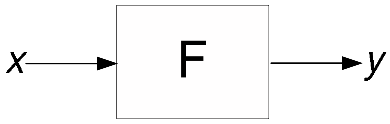
(e) If the input signal x, characterized by $x(n) = 1, \forall n$, is applied to the filter, the output signal y is $y(n) = 1, \forall n$.

(f) It is known that

$$\lim_{z \to 0} z^2 \widehat{F}(z) = 1.$$

Determine, and provide a well-labeled plot of, the impulse response f.

MT1.3 (50 Points) Consider a *BIBO stable* discrete-time LTI filter $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:



The filter has impulse response f, frequency response F, and rational transfer function $\widehat{F}$.

In particular, the transfer function is given by

$$\widehat{F}(z) = \frac{z^4 - 1}{z^4 - (0.99)^4} \,.$$

(a) Provide a well-labeled pole-zero diagram for $\widehat{F}$.

(b) Select the strongest true assertion from the list below. Explain your selection succinctly, but clearly and convincingly.

(i) The system must be *causal*.

(ii) The system could be *causal*, but does not have to be.

(iii) The system cannot be *causal*.

(c) Determine the linear, constant-coefficient difference equation governing the input-output behavior of the filter F.

(d) Provide a well-labeled delay-adder-gain block diagram implementation of the filter. Your implementation must employ the minimal number of delay elements possible.

You may coalesce multiple delays into one block, e.g., use a single z^{-N} delay block instead of drawing N one-sample delay blocks in cascade.

(e) For the frequency range $-\pi < \omega \leq +\pi$, provide a well-labeled sketch of $|F(\omega)|$, the magnitude response of the filter F.

Is the filter F low-pass, band-pass, high-pass, notch, anti-notch[1], or none of these?

(f) Determine the response y of the filter F, if the input signal x is

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{m=-\infty}^{\infty} \delta(n - 4m).$$

(g) Consider a DT-LTI filter G whose impulse response is

$$\forall n \in \mathbb{Z}, \quad g(n) = \left[(0.99)^4\right]^n u(n) - \left[(0.99)^4\right]^{n-1} u(n-1).$$

Determine an explicit relationship between the impulse responses f and g.

[1] An anti-notch filter notches up, instead of down, at one or more frequencies, and is flat elsewhere.

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Problem Name	Points 10	Your Score
1	10	
2	45	
3	50	
Total	**115**	