1. **15 points** Find the FT of $x$, and sketch the real and imaginary parts of $X(\omega)$, where

$$\forall t, x(t) = \prod(t) \ast \prod(t) \ast \sum_{-\infty}^{\infty} \delta(t - 8n)$$

Here $\prod(t) = 1$ for $|t| \leq \frac{1}{2}$ and 0, else. In your sketch carefully mark the relevant frequencies and magnitudes.
2. **15 points**

(a) Find and sketch the FT of

\[ x(t) = \left( \frac{\sin \pi t}{\pi t} \right)^2 e^{-j2\pi 10t}. \]

(b) Use Parseval’s theorem to find the energy in the signal \( x, \int_{-\infty}^{\infty} [x(t)]^2 dt. \)
3. **10 points** The following statements are either TRUE or FALSE. If you believe a statement is true, outline a BRIEF PROOF. If you believe it is false, provide a BRIEF COUNTEREXAMPLE.

(a) If \( x(t), t \in \text{Reals}, \) is a real valued signal, its Fourier transform \( X(f), f \in \text{Reals}, \) is also real valued.

(b) If \( x(t), y(t), t \in \text{Reals}, \) are real-valued signals and \( (x * y)(t) = 0, \forall t \in \text{Reals}, \) then either \( x \) or \( y \) is identically zero.

(c) If \( x(t), t \in \text{Reals}, \) is a real-valued, baseband signal with bandwidth \( W \) Hz, then the signal \( y(t) = 2x^2(t) + 3x^4(t), t \in \text{Reals}, \) has bandwidth at most \( 4W \)Hz.

(d) If \( x(t), t \in \text{Reals} \) is a real-valued, band-limited signal with bandwidth \( W \) Hz, then the signal \( y(t) = x, t \in \text{Reals}, \) has a bandwidth \( W^2 \) Hz.

(e) If \( x, y \) are real-valued signals with bandwidth \( W_x, W_y \) Hz, respectively, then a the signal \( x + y \) has bandwidth \( W_x + W_y \) Hz.
4. **15 points** The following statements are either TRUE or FALSE. If you believe a statement is true, outline a BRIEF PROOF. If you believe it is false, provide a BRIEF COUNTEREXAMPLE.

(a) The system that takes as input a signal $m$ and produces its Hilbert transform $\hat{m}$ as output is an LTI system.

(b) The SSB-USB modulator which takes as input a signal $m(t), t \in Reals$, and produces as output the modulated signal $x(t), t \in Reals$, is a linear system.

(c) The narrow-band FM system which takes as input the continuous-time signal $m$ and produces as output the modulated signal $x$, is a linear system.

(d) The AM-DSB modulator is a time-invariant system.

(e) The signal $\forall t, x(t) = \cos(2\pi f_c t + \cos(2\pi f_m t)))$ $(f_m \neq 0)$ has infinite bandwidth.

(f) Is it possible to recover the signals $A$ and $\theta$ from the narrowband signal $\forall t, x(t) = A(t) \cos(2\pi f_c t + \theta(t))$. 
5. **20 points** Figure 1 is a block diagram of vestigial sideband (VSB) modulation/demodulation.

![Block Diagram of VSB Modulation/Demodulation](image)

**Figure 1:** The VSB modulation-demodulation scheme of problem 5

The baseband signal $m(t)$ has FT $M(\omega)$ as shown, with bandwidth $B$ rad/sec. It modulates the carrier $\cos(\omega_c t)$ ($\omega_c \gg B$) to produce the signal $u(t)$, which is passed through the VSB filter, whose frequency response $H(\omega)$ is shown. The result is the transmitted signal $v(t)$. The coherent receiver multiplies $v(t)$ by the carrier to produce $w(t)$, which is then passed through a low pass filter (LPF) (with bandwidth $B$) to obtain the signal $x(t)$.

(a) Obtain mathematical expressions for the FT of $u(t)$, $v(t)$, and $w(t)$. Sketch the FTs, and carefully mark the relevant magnitudes and frequencies.

(b) Show that $x = (1/4)m$ if the VSB filter satisfies

$$H(\omega + \omega_c) + H(\omega - \omega_c) = 1, \text{ for } |\omega| \leq B.$$
6. **10 points** Figure 2 is a block diagram of a digital communication system. The digital channel accepts

![Figure 2: The communication system for problem 6](image)

at its input port any symbol from \{a, b, c, d, e\} and delivers it at its output port. The channel can accept one symbol every $2\mu$ sec.

(a) What is the baud rate of the channel in symbols/sec? What is its capacity in bits/sec?

(b) A binary source $m$ produces data at 1 Mb/sec. (1 Mb is one million bits.) Is the rate of the source smaller than the capacity? If it is, construct a “coder” that maps the binary source $m$ into a sequence of symbols $x$, and a “decoder” that maps $x$ into a binary sequence $m'$ such that $m' = m$. 


7. **15 points** \( m \) is a **complex-valued** signal with bandwidth \( B_m \) rad/sec whose real and imaginary parts are \( m_1, m_2 \) respectively. Let \( M(\omega), M_1(\omega) \) and \( M_2(\omega) \) be the FT of \( m, m_1, \) and \( m_2 \) respectively.

(a) Find \( M_1 \) and \( M_2 \) in terms of \( M \). Show that the bandwidth of \( m_1, m_2 \) is at most \( B_m \).

(b) Design a modulation and demodulation scheme that can transmit \( m_1 \) and \( m_2 \) over a channel with bandwidth \( 2B_m \) centered at frequency \( \omega_c \) rad/sec.

(c) Give a brief mathematical argument to show that the transmitted signal is within the channel bandwidth, and that the receiver can recover both signals.