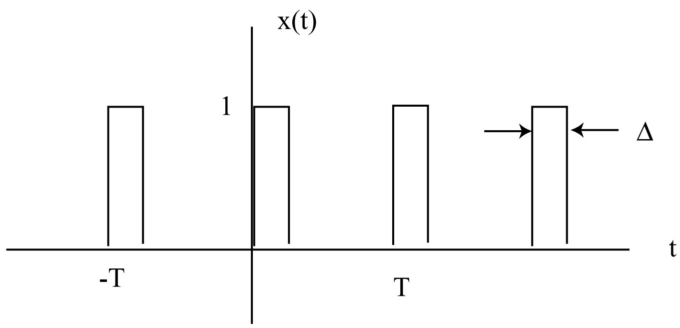

Figure 1: Periodic signal for problem 1

EECS 120. Final Exam Solution, May 19, 2000. 12.30-3.30 pm.

1. **20 points** Consider the periodic signal x of Figure 1.

   (a) Evaluate the coefficients X_k in the Fourier series

   $$\forall t, \quad x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}.$$

   (b) What is ω_0? State the units.

   (c) Evaluate

   $$\sum_{k=-\infty}^{\infty} |X_k|^2.$$

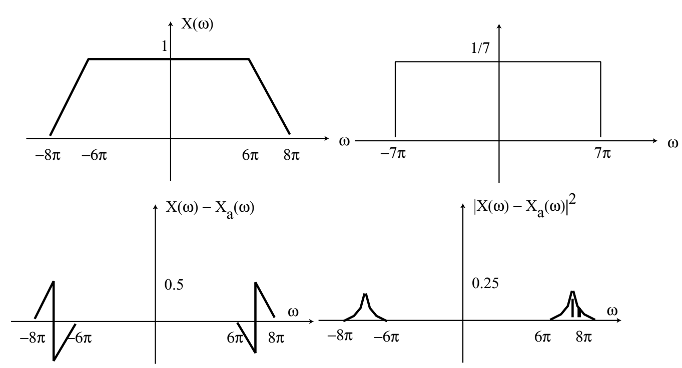

Figure 2: Signal and low pass filter for problem 2

2. **20 points** Consider the signal x with Fourier Transform given in the left of Figure 2.

(a) What is the bandwidth of this signal in rad/sec and in Hz. What is the lowest sampling frequency in Hz that allows exact reconstruction of the signal from its samples.

(b) Suppose x is sampled at 7 Hz and the sampled signal x_s is the product of x and $f_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - n/7)$. Sketch the Fourier Transform X_s of x_s.

(c) Let x_a be the output when x_s is passed through the low pass filter shown on the right in the figure. Sketch the Fourier transform X_a of x_a and determine the squared error $\int_{-\infty}^{\infty} |x(t) - x_a(t)|^2 dt$, without a lot of computation.

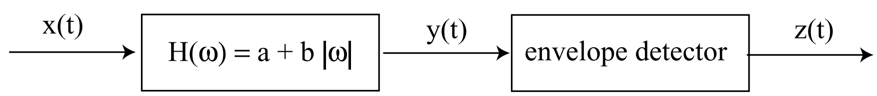

Figure 3: Demodulation scheme in Problem 3

3. **20 points** Consider a NB signal of the form

$$x(t) = \cos(\omega_c t + \theta(t)).$$

Assume $X(\omega)$ is zero except where $||\omega| - \omega_c| < 2\pi W|$ and $W << \omega_c$. Find y, z in the arrangement of Figure 3.

(Hint. In $H(\omega)$ you may use $|\omega| = (j\omega)(-j\text{sgn}\omega)$, where $\text{sgn}(w)$ is the function equal to +1 if $w \geq 0$ and -1 if $w < 0$.)

4. **20 points** The step response of an LTI system is given by

$$s(t) = \begin{cases} 0, & t \leq 0 \\ 1 - 0.5e^{-t} + 0.5e^{-2t}, & t > 0 \end{cases}$$

(a) Find its impulse response, transfer function, and frequency response.

(b) Find its steady state response to the input

$$\forall t, \quad x(t) = \cos(\omega t)u(t).$$

(c) Find its response to the input

$$\forall t, \quad x(t) = \cos(\omega t).$$

5. **20 points** Consider the linear vector differential equation system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c'x(t) + du(t) \end{aligned}$$

where $x = (x_1, x_2)' \in R^2$ and

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad c' = [1 \quad 0] \quad d = 0.$$

(a) Find the transfer function $H(s) = Y(s)/U(s)$.

(b) The Laplace transform of the matrix-valued function $e^{tA}u(t)$ is the matrix $[sI - A]^{-1}$. Calculate this matrix and then take its inverse Laplace transform to calculate e^{tA}.

(c) Suppose the initial state is $x = (1, 1)$. Find the zero-input response for this initial state.

(d) Find the zero-state step response.

(e) Find the response when the initial state is $(1, 1)$ and the input is a unit step.

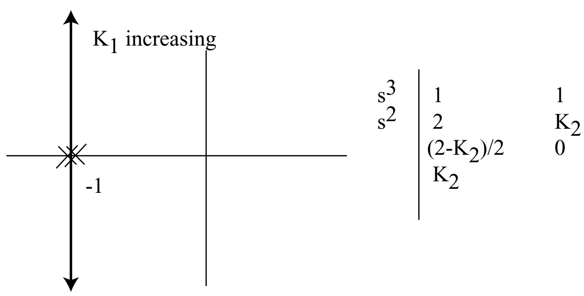

Figure 4: Root locus in Problem 6

6. **20 points** The open loop plant has transfer function $H(s) = 1/(s^2 + 2s + 1)$. Place the plant in a closed loop using a PI controller $K_1 + K_2/s$.

   (a) Take $K_2 = 0$, and plot the root locus as K_1 varies. For what values of $K_1 \geq 0$ is the closed loop system stable? What is the steady state error to a step input as a function of K_1?

   (b) Take $K_1 = 0$, and use the Routh-Hurwitz criterion to find the values of $K_2 > 0$ such that the closed loop system is stable. What is the steady-state error for step inputs as a function of K_2?

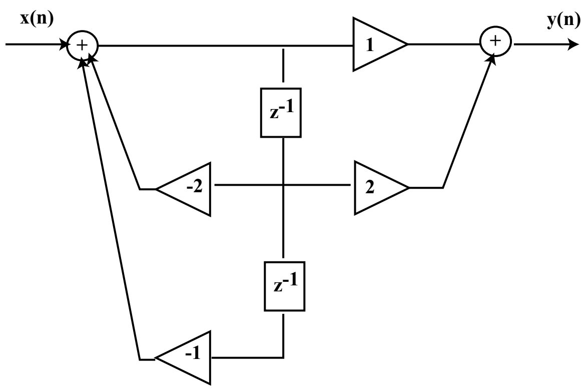

Figure 5: Direct form realization for problem 7

7. **20 points** Consider the difference equation

$$y(k) + 2y(k-1) + y(k-2) = x(k) + 2x(k-1), k = 0, 1, 2, ...$$

(a) Find the transfer function $H(z)$. Is the system BIBO stable?

(b) Find the impulse response, assuming zero initial conditions.

(c) Obtain a direct form realization of the difference equation using only two delay elements.

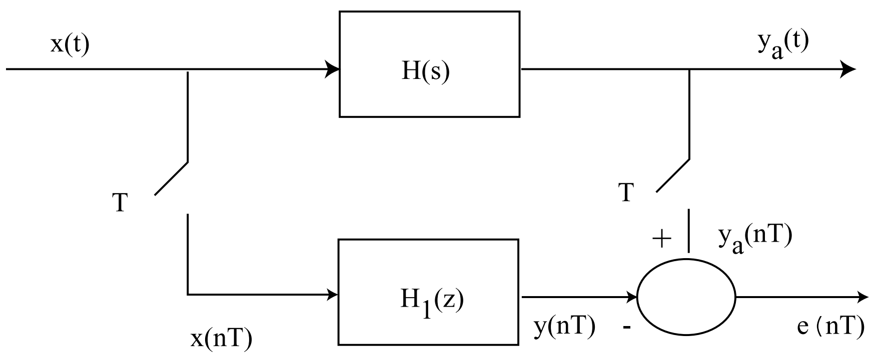

Figure 6: System for problem 8

8. **20 points**

   Consider the analog transfer function $H(s) = 1/(s^2 + 5s + 6)$.

   (a) Consider the scheme of Figure 6. Suppose $x(t) = u(t)$. Find $H_1(z)$ so that $e(nT) \equiv 0$, for all n. This is the step-invariant filter.

   (b) Suppose $x(t) = \delta(t)$. Find $H_1(z)$ so that $e(nT) \equiv 0$. In this case assume that $x(nT)$ is the Kronecker delta. This is the impulse-invariant filter.