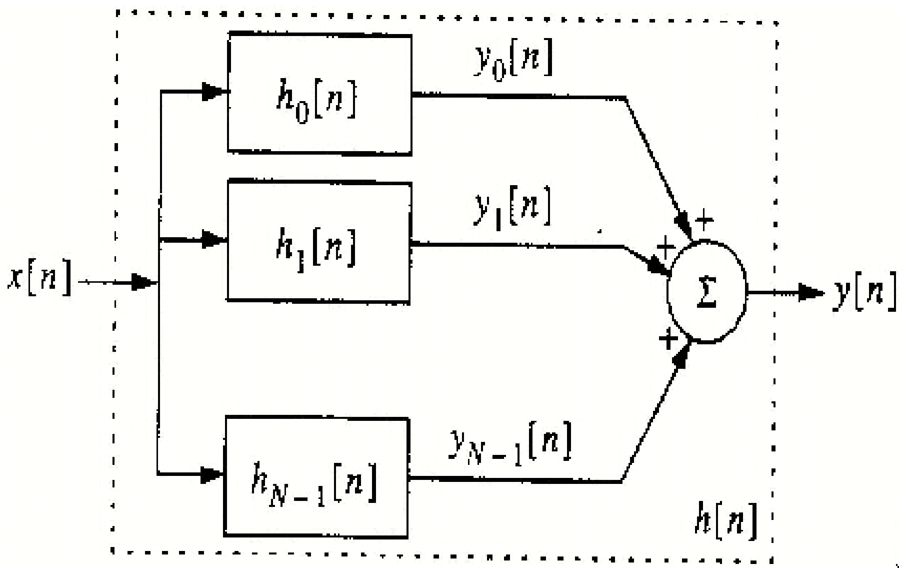
EE120: Fall 99 --Midterm2 Prof. J.M. Kahn

Problem #1

(25pts.) Consider a DT LTI system having input x[n], impulse response h[n] and output y[n]. The system is composed
paralled interconnection of N DT LTI systems having impulse responses h(sub k)[n], k = 0,...,N-1.

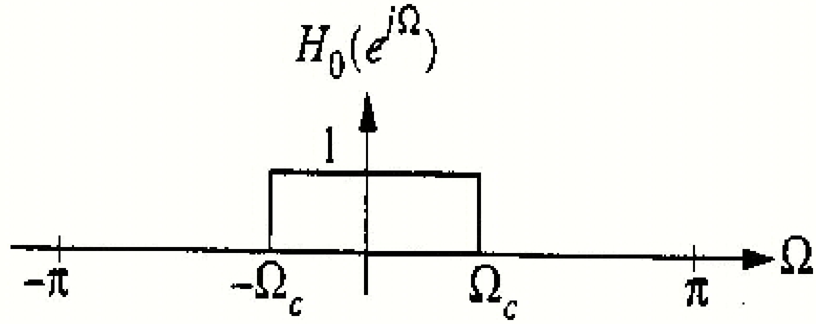


For any k, $h_k[n]$ is related to $h_0[n]$ by $h_k[n] = e^{j2\pi nk/N} h_0[n]$.

For any k, h(sub k)[n] is related to h(sub 0)[n] by h(sub k)[n] = e^(j*2pi*n*k/N)*h(sub 0)[n].

a) (5 pts.) Let Hk(e^(j*omega)) and Ho(e^(j*omega)) denote the DTFTs of h(sub k)[n] and h(sub 0)[n], respectively. F
an expression for Hk(e^(j*omega)) in terms of Ho(e^(j*omega)).

In parts (b) and (c), let h(sub 0)[n] be an ideal lowpass filter with the frequency response Ho(e^(j*omega)) as shown below for the range -pi <= omega < pi. The cutoff frequency is OMEGAc, where 0 < OMEGAc < pi.

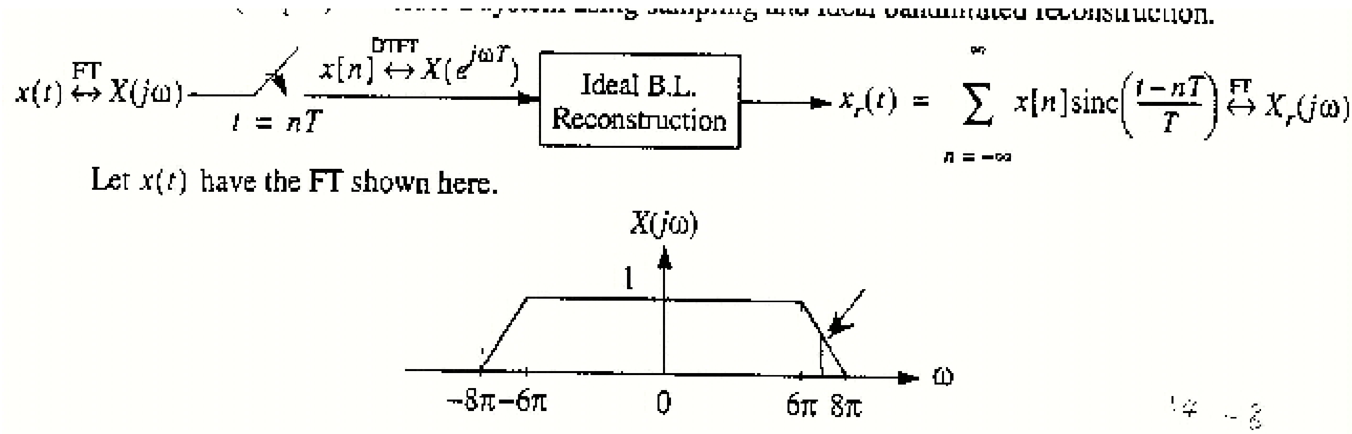


(b) (10 pts.) Sketch H(sub 1)(e^(j*omega)) and H(sub (N-1))(e^(j*omega)) for -pi <= omega < pi, labeling the vertical
horizontal axes of the plots.

(c) (10 pts.) Determine, in terms of N, the value of OMEGAc, 0 < OMEGAc < pi, such that y[n] = x[n].

Problem #2

(40 pts.) Consider a system using sampling and ideal bandlimited reconstruction.



(a) (5 pts.) What is the largest T such that x(sub r)(t) = x(t)?

In parts (b), (c) and (c), assume that T = 1/7.

(b) (10 pts.) Sketch the DTFT of the sampled signal, X(e^(j*w*t). Label the vertical and horizontal axes of your plot.

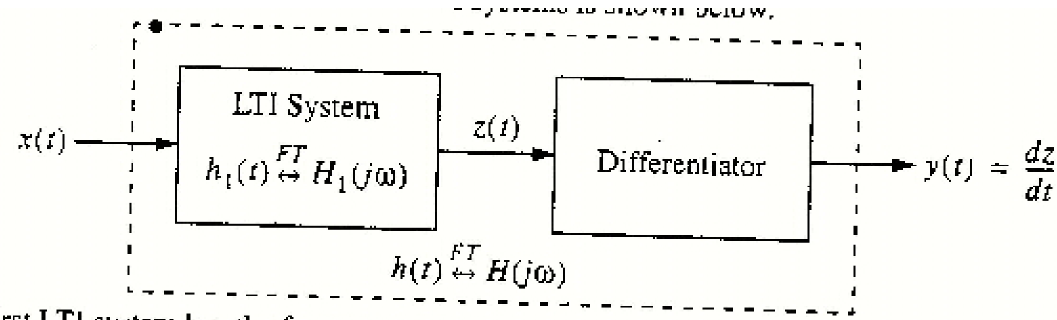
(c) (10 pts.) Sketch the FT of the reconstructed signal, Xr(jw). Label the vertical and horizontal axes of your plot.

$$\varepsilon = \int_{-\infty}^{\infty} |x(t) - x_r(t)|^2 dt.$$

(d) (15 pts.) Use Parseval's identity to calculate the squared error

Problem #3

(35 pts.) A cascade of two LTI systems is shown below.



The first LTI system has the frequency response:

$$H_1(j\omega) = \begin{cases} e^{-j10\omega} & |\omega| \leq 10\pi \\ 0 & |\omega| > 10\pi \end{cases}$$

(a) (5 pts.) Find an expression for H(jw), the frequency response of the overall system enclosed in the dashed box.

(b) (10 pts.) Sketch |H(jw)| and arg {H(jw)}, labeling the vertical and horizontal axes. (c) (15 pts.) Find an expression f h(t), the impulse response of the overall system enclosed in the dashed box.

(d) (5 pts.) Let the input be x(t) = sin(pi/20 *t). Find an expression for the output y(t).