EE120 Fall, 1998

Midterm #2 Solutions

1)

a) Let $\omega_0 = (2\pi) / T$

$$c(t) \overset{Fs}{\longleftrightarrow} c[k] = (\tau/T)\ \mathrm{sinc}\,(k\tau/T) = [(\omega_0\tau) / (2\pi)]\ \mathrm{sinc}\,[(k\omega_0\tau) / (2\pi)]$$

$$c(j\omega) = 2\pi \sum_{k=-\infty}^{+\infty} c[k]\ \delta(\omega - k\omega_0)$$

$$= (\omega_0\tau) \sum_{k=-\infty}^{+\infty} \mathrm{sinc}\,[(k\omega_0\tau) / (2\pi)]\ \delta(\omega - k\omega_0)$$

$$= (2\pi\tau / T) \sum_{k=-\infty}^{+\infty} \mathrm{sinc}\,[(k\tau/T)]\ \delta(\omega - k2\pi/T)$$

b)

$$y(t) = m(t) \cdot c(t)$$

$$y(j\omega) = 1/(2\pi)\, M(j\omega) * C(j\omega)$$

$+\infty$

$$= [(\omega_0 \tau) / (2\pi)] \sum_{k=-\infty}^{+\infty} \text{sinc}\,[(k\omega_0 \tau) / (2\pi)]\, M(j(\omega - k\omega_0))$$

$$= (\tau / T) \sum_{k=-\infty}^{+\infty} \text{sinc}\,[(k\tau)/T]\, M(j(\omega - k2\pi/T))$$

c)

$(2\pi / T) = \omega_0 = 2\pi$

$\tau / T = (\omega_0 \tau) / (2\pi) = 1/4$

$$y(j\omega) = 1/4 \sum_{k=-\infty}^{+\infty} \text{sinc}\,(k/4)\, M(j(\omega - k2\pi))$$

d) Want $S(j\omega) = \frac{1}{2}\left(M(j(\omega - 2\pi)) + M(j(\omega + 2\pi))\right)$

$H_0 = ½ / ¼ \operatorname{sinc}(¼) = 2 / \operatorname{sinc}(¼)$

2)

a) $S(j\omega) = ½ \left(M(j(\omega - 4\pi)) + M(j(\omega - 4\pi))\right)$

b)

$$S(e^{j\omega T}) = 1/T \sum_{k=-\infty}^{+\infty} S(j(\omega - k2\pi/T)) = \sum_{k=-\infty}^{+\infty} S(j(\omega - k2\pi))$$

c) Let $h(t) = \text{sinc}(t/T) \leftrightarrow H(j\omega) = T \quad \text{if } |\omega| \le \pi/T$

$0 \text{ if } |\omega| > \pi/T$

$$y(t) = \sum_{k=-\infty}^{+\infty} m[n]\, h(t - nT) = h(t) * \sum_{k=-\infty}^{+\infty} m[n]\, \delta(t - nT)$$

$$= h(t) * \left[m(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t - nT)\right]$$

$$y(j\omega) = H(j\omega) \cdot \left[1/(2\pi)\, M(j\omega) * (2\pi/T) \sum_{k=-\infty}^{+\infty} \delta(\omega - k2\pi/T)\right]$$

$$= (1/T)\, H(j\omega) \cdot \sum_{k=-\infty}^{+\infty} M(j(\omega - k2\pi/T))$$

$$= M(j\omega)$$

3)

a) $Y_d(e^{j\omega}) = (1 / T) \sum_{k=-\infty}^{+\infty} Y_c(j(\omega - k2\pi / T))$

b) $Y_c(j\omega) = H_c(j\omega) \cdot X_c(j\omega)$

c) Let $h_0(t) = \mathrm{sinc}(t/T)$

$H_0(j\omega) = T$ if $|\omega| \le \pi/T$

$= 0$ if $|\omega| > \pi/T$

$X_c(t) = \sum_{k=-\infty}^{+\infty} d[n]\, h(t - nT) = h_0(t) * \sum_{k=-\infty}^{+\infty} X_d[n]\, \delta(t - nT)$

$X_c(j\omega) = H_0(j\omega) \cdot X_d(e^{j\omega T})$

$= T X_d(e^{j\omega T})$ if $|\omega| \le \pi/T$

0 if $|\omega| > \pi/T$

d) $Y_d(e^{j\omega}) = (1 / T) \sum^{+\infty} H_c(j\omega) \cdot H_0(j\omega) \cdot X_d(e^{j\omega T})$

$k = -\infty$

where H_c is aperiodic; H_0 selects $|\omega| \le \pi / T$; X_d with $(2\pi / T)$ is periodic

$$= X_d(e^{j\omega T}) \cdot (1 / T) \sum_{k=-\infty}^{+\infty} H_c(j\omega) \cdot H_0(j\omega)$$

$$H_d(e^{j\omega T}) = \frac{Y_d(e^{j\omega T})}{X_d(e^{j\omega T})}$$

$$= (1 / T) \sum_{k=-\infty}^{+\infty} H_c(j\omega) \cdot H_0(j\omega)$$

$H_d(e^{j\omega T})$ is the periodic extension of a bandlimited version of $H_c(j\omega)$

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