University of California at Berkeley
Department of Electrical Engineering and Computer Sciences
Professor J.M. Kahn, EECS 120, Fall, 1997
Final Examination, Wednesday, December 17, 1997, 5-8 pm

NAME: ____

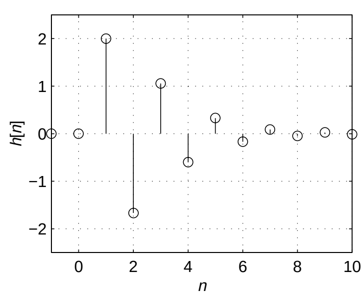
1. The exam is closed-book. You are permitted to use three, two-sided pages of notes. No calculators are permitted.
2. Do all work in the space provided. If you need more room, use the back of previous page.
3. Indicate your answer clearly by circling it or drawing a box around it.

Problem	1	2	3	4	5	6	TOTAL
Points	75	30	30	25	30	10	200
Score							

Problem 1 (75 pts.) A LTI discrete-time system with input $x[n]$ and output $y[n]$ has an impulse response:

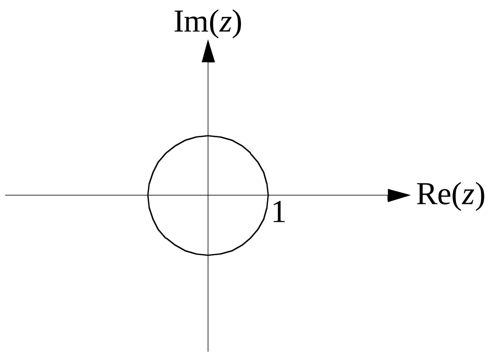
$$h[n] = 12\left[\left(-\frac{1}{3}\right)^n - \left(-\frac{1}{2}\right)^n\right]u[n].$$

A sketch of $h[n]$ is shown here.



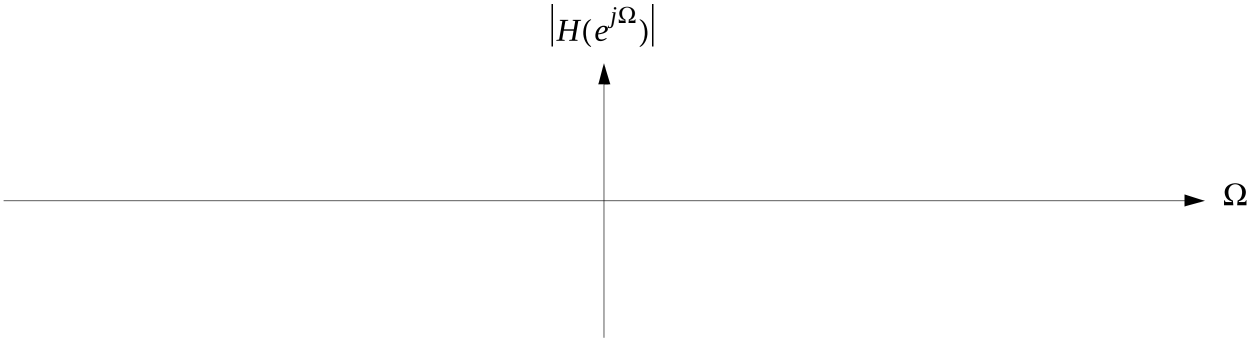
(a) (5 pts.) Give a closed-form expression for the system transfer function $H(z)$.

(b) (5 pts.) Find the poles and zeros of $H(z)$, and sketch them on the z-plane below.



(c) (5 pts.) Find the output $y[n]$ when input is $x[n] = 3^n, \forall n$.

(d) (5 pts.) Clearly the frequency response $H(e^{j\Omega})$ exists. Sketch the magnitude response $|H(e^{j\Omega})|$, labeling the vertical and horizontal axes of your plot.

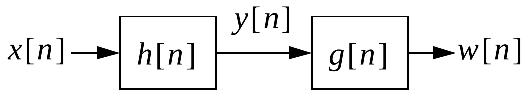


(e) (10 pts.) Evaluate $\int_{0}^{2\pi} H(e^{j\Omega})e^{-j\Omega}d\Omega$ without explicitly using the expression for $H(e^{j\Omega})$.

(f) (10 pts.) Evaluate $\int_{-\pi}^{\pi} \left|H(e^{j\Omega})\right|^2 d\Omega$ without explicitly using the expression for $H(e^{j\Omega})$.

(g) (15 pts.) Find an expression for the step response $y_s[n]$.

The step response exhibits overshoot, which is troublesome in many applications. It is proposed to compensate for the overshoot by cascading with another system $g[n]$, as shown below.



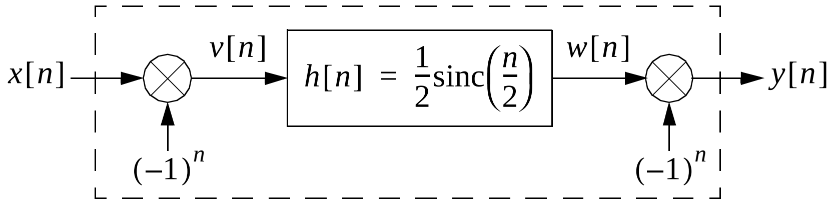
The system $g[n]$ is described by the difference equation:

$$w[n] = ay[n] + by[n-1] + cy[n-2].$$

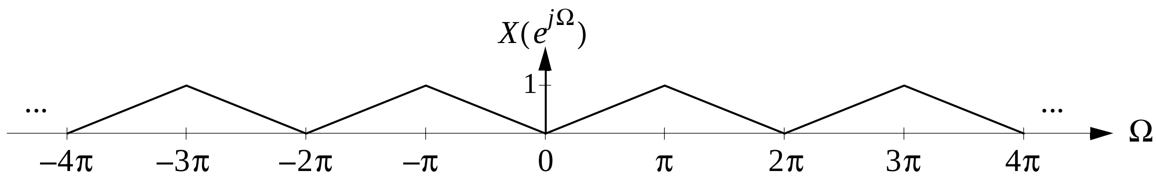
(h) (15 pts.) Find values of the constants a, b, and c such that the cascaded system (with input $x[n]$ and output $w[n]$) has a step response that is simply a delayed unit step, $w_s[n] = u[n-1]$.

(i) (5 pts.) Sketch a realization of the system $g[n]$.

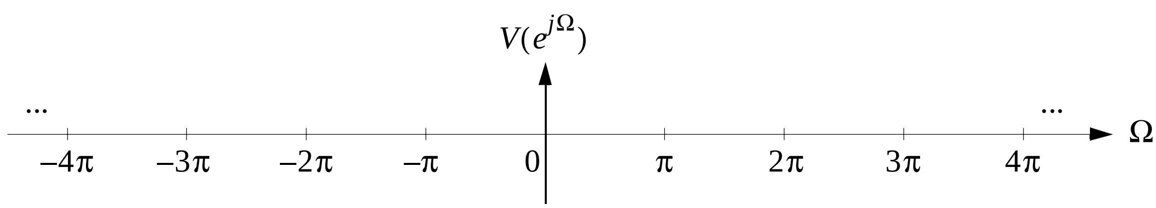
Problem 2 (30 pts.) Consider the discrete-time system enclosed in the dashed box, which has input $x[n]$ and output $y[n]$.

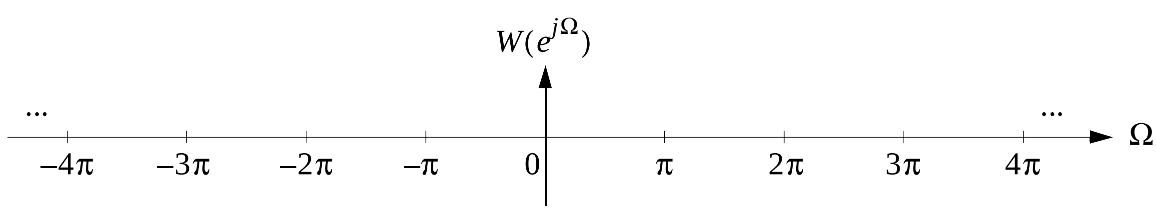


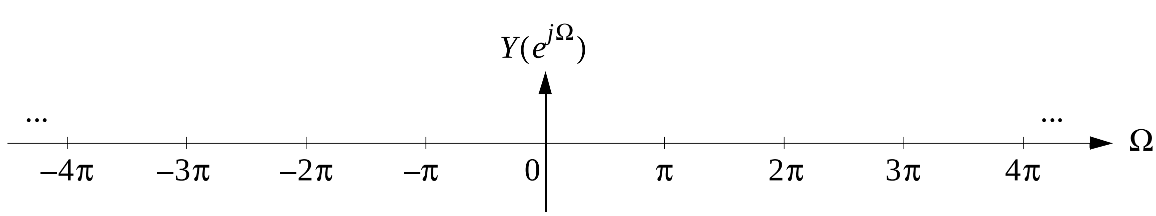
Suppose that the input $x[n]$ has the discrete-time Fourier transform $X(e^{j\Omega})$ shown below.



(a) (15 pts.) Sketch $V(e^{j\Omega})$, $W(e^{j\Omega})$, and $Y(e^{j\Omega})$, which are the discrete-time Fourier transforms of $v[n]$, $w[n]$, and $y[n]$, respectively.

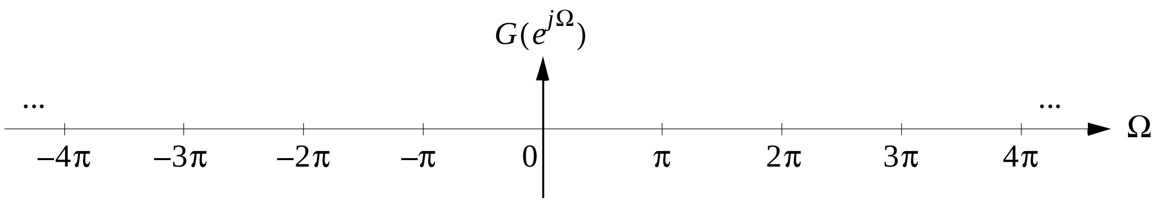






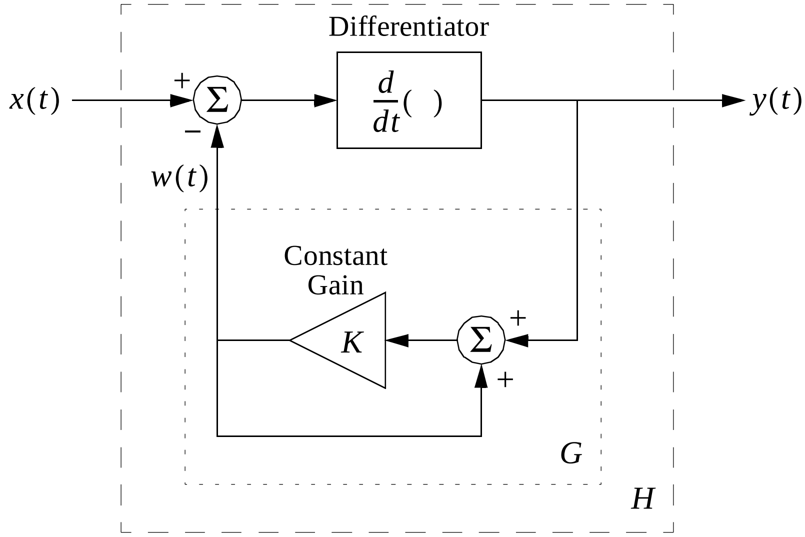
Evidently, the system enclosed in the dashed box is equivalent to a LTI system $g[n] \leftrightarrow G(e^{j\Omega})$.

(b) (5 pts.) Sketch the frequency response $G(e^{j\Omega})$.



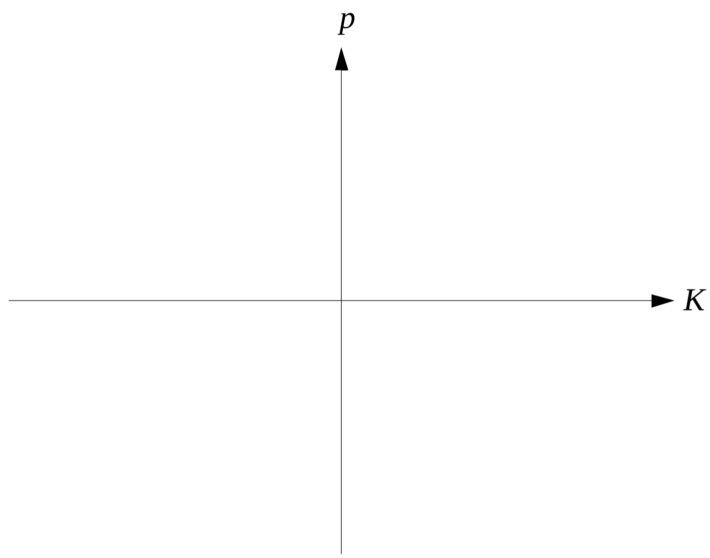
(c) (10 pts.) Give an expression for the impulse response $g[n]$.

Problem 3 (30 pts.) Consider the LTI continuous-time system H enclosed in the outer dashed box, which has input $x(t)$ and output $y(t)$. K is a real constant, $-\infty < K < \infty$.

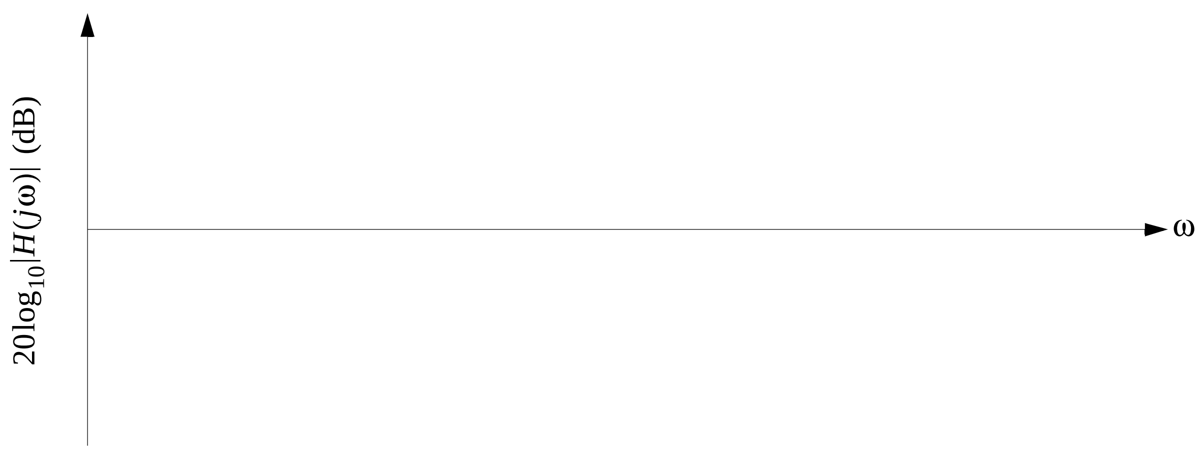


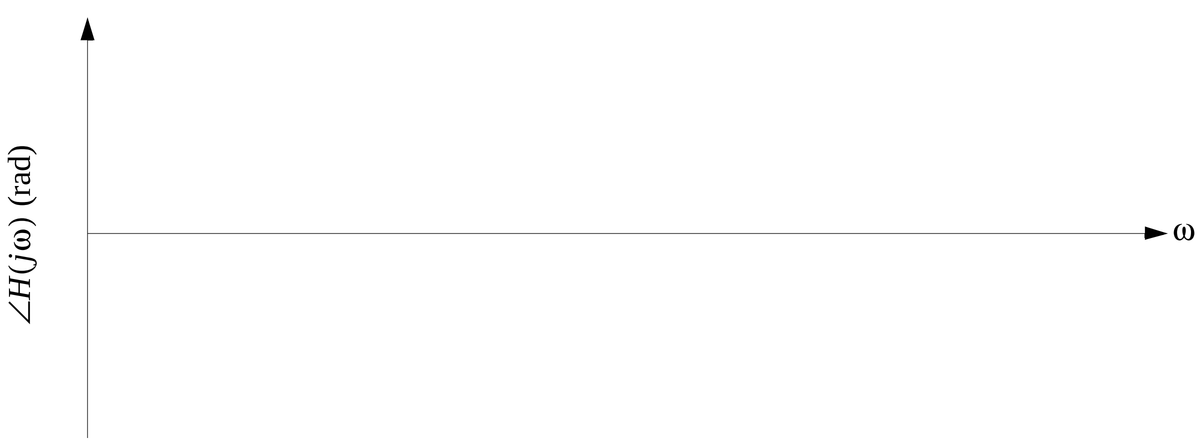
(a) (10 pts.) Find the transfer function $H(s) = Y(s)/X(s)$. *Hint*: first consider G, the system enclosed in the inner dotted box, which has input $y(t)$ and output $w(t)$. It has transfer function $G(s) = W(s)/Y(s)$.

(b) (5 pts.) $H(s)$ has one zero at $s = z$, and one pole at $s = p$. Find z and p in terms of K. Sketch p as a as a function of K, labeling the vertical and horizontal axes of your plot. For what values of K is the system H stable?



(c) (15 pts.) Assume that $K = 1/3$. Make Bode plots of the magnitude and phase of the frequency response $H(j\omega)$, labeling the horizontal and vertical axes carefully.





Problem 4 (25 pts.) As studied in class, Parseval's identity for the continuous-time Fourier transform states that for $x(t) \leftrightarrow X(\omega)$:

$$\int_{-\infty}^{\infty} x^*(t)x(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)X(\omega)d\omega .$$

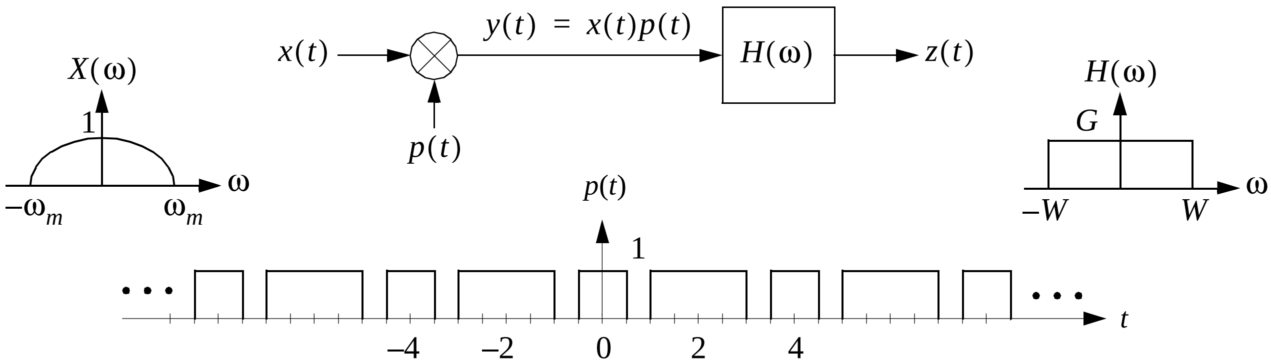
(a) (15 pts.) Consider $x_1(t) \leftrightarrow X_1(\omega)$ and $x_2(t) \leftrightarrow X_2(\omega)$. Prove a generalization of the identity:

$$\int_{-\infty}^{\infty} x_1^*(t)x_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1^*(\omega)X_2(\omega)d\omega .$$

(b) (10 pts.) Let $x(t)$ be a real signal, and let $\hat{x}(t)$ be its Hilbert transform. Use the result of part (a) to evaluate the integral:

$$\int_{-\infty}^{\infty} x^*(t)\hat{x}(t)dt .$$

Problem 5 (30 pts.) Consider the following system.



The signal $x(t)$ is bandlimited to $|\omega| \le \omega_m$, It is multiplied by $p(t)$, the periodic signal shown. The product, $y(t)$, is passed through the ideal lowpass filter $H(\omega)$, which has cutoff frequency W and gain G.

(a) (10 pts.) Since $p(t)$ is periodic with period $T_0 = 4$, it can be represented as a Fourier series:

$$p(t) = \sum_{n = \infty}^{\infty} P_n e^{\frac{jn\pi t}{2}} .$$

Without explicitly calculating the P_n, find an expression for $Y(\omega)$, the Fourier transform of $y(t)$, in terms of $X(\omega)$ and the P_n.

(b) (20 pts.) State the conditions on ω_m, W and G such that $z(t) = x(t)$. Be as specific as possible, replacing variables by specific numbers when possible.

Problem 6 (10 pts.) A discrete-time LTI system has transfer function:

$$H(z) = \frac{1}{1 - z^{-17} - z^{-19}}.$$

The input is $x[n] = a^n u[n]$. Assuming $y[n] = 0, n < 0$, find $y[15]$.