# University of California at Berkeley <br> Department of Electrical Engineering and Computer Sciences <br> Professor J.M. Kahn 

## EECS 120

Midterm 1
Monday, October 14, 1996, 2:10-3:10 pm

Name: $\qquad$

1. The exam is open book and open notes.
2. Pace yourself. Don't spend too much time on any one problem.
3. Do all work in the space provided. If you need more room, use the back of previous page.
4. Indicate your answer clearly by circling it or drawing a box around it.
5. Think carefully about the problem before you begin to write.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 15 |  |
| 2 | 30 |  |
| 3 | 35 |  |
| 4 | 20 |  |
| TOTAL: | 100 |  |

Problem 1 (15 pts.) The table below presents three systems that have input $x(t)$ and output $y(t)$. Please specify (yes or no) whether each system is linear, time-invariant, memoryless, and causal. You needn't justify your answers.

| System | Linear? | Time- <br> Invariant? | Memoryless? | Causal? |
| :---: | :---: | :---: | :---: | :---: |
| $y(t)=x(t) \cdot \cos t+2$ |  |  |  |  |
| $y(t)=x(2 t)$ |  |  |  |  |
| $y(t)=x(t) \otimes u(t)$ |  |  |  |  |

Problem 2 ( 30 pts.) Consider a system with input $x(t)$ and output $y(t)$ that is governed by the differential equation:

$$
\dot{y}(t)+y(t)=\dot{x}(t) .
$$

(a) (10 pts.) Find the frequency response $H(\omega)$.
(b) (10 pts.) Sketch the magnitude $H(\omega)$ and phase $\angle H(\omega)$. Be sure to label the vertical and horizontal axes of your plots.

(c) (5 pts.) Suppose the input to the system is $x(t)=\sin t$. What is $y(t)$ ?
(d) (5 pts.) Suppose the input to the system is $x(t)=2$. What is $y(t)$ ?

Problem 3 ( 35 pts.) A periodic signal is described by the expression:

$$
x(t)=\sum_{n=-\infty}^{\infty}\left[4 \Pi\left(\frac{t-n}{1 / 4}\right)-2 \Pi\left(\frac{t-n-0.5}{1 / 2}\right)\right] .
$$

(a) (10 pts.) Sketch $x(t)$, labeling the horizontal and vertical axes.

(b) (15 pts.) Specify the fundamental frequency $\omega_{0}$ and find an exponential Fourier series representation for $x(t)$. Hint: use Fourier series you already know!
(c) (10 pts.) $x(t)$ is passed through an ideal lowpass filter $H_{1}(\omega)$ having bandwidth $W$, yielding $y(t)$. Specify the range of values of $W$ such that $y(t)=0$.


Problem 4 (20 pts.) A first-order highpass filter with input $x(t)$ and output $y(t)$ is shown below.


The system impulse response is $h(t)=\delta(t)-e^{-t} u(t)$. Given an input $x(t)=r(t)=u(t) \otimes u(t)$, find an expression for the output $y(t)$. Sketch $y(t)$, labeling the horizontal and vertical axes.


