University of California at Berkeley
Department of Electrical Engineering and Computer Sciences
Professor J.M. Kahn, EECS 120, Fall, 1996
Final Examination, Wednesday, December 18, 1996, 5-8 pm

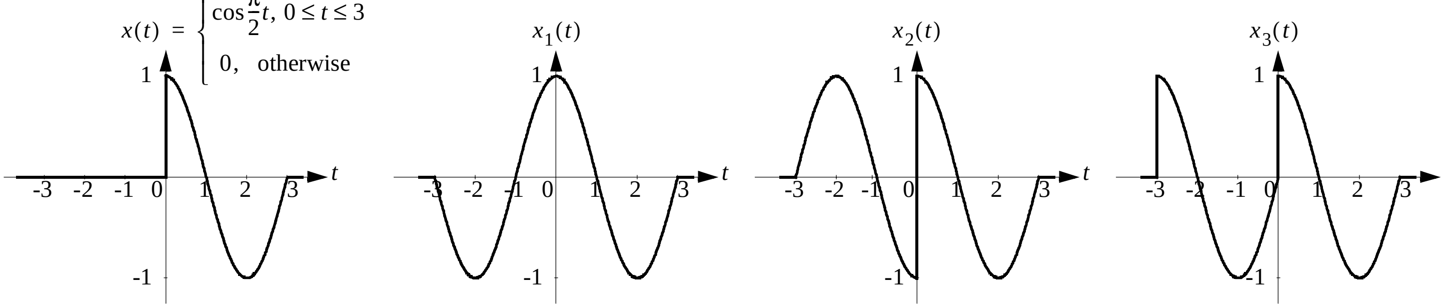
NAME: ____

1. The exam is open book and open notes.
2. Do all work in the space provided. If you need more room, use the back of previous page.
3. Indicate your answer clearly by circling it or drawing a box around it.

Problem	1	2	3	4	5	6	TOTAL
Points	35	15	25	30	35	60	200
Score							

Problem 1 (35 pts.) Consider the four signals $x(t), x_1(t), x_2(t), x_3(t)$, which have Fourier transforms $X(\omega), X_1(\omega), X_2(\omega), X_3(\omega)$, respectively.

$$x(t) = \begin{cases} \cos\frac{\pi}{2}t, & 0 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$$



(a) (10 pts.) Use an “X” to indicate any symmetry property that pertains.

Fourier Transform	Property			
	Purely Real	Purely Imaginary	Even function of ω	Odd function of ω
$X(\omega)$				
$X_1(\omega)$				
$X_2(\omega)$				
$X_3(\omega)$				

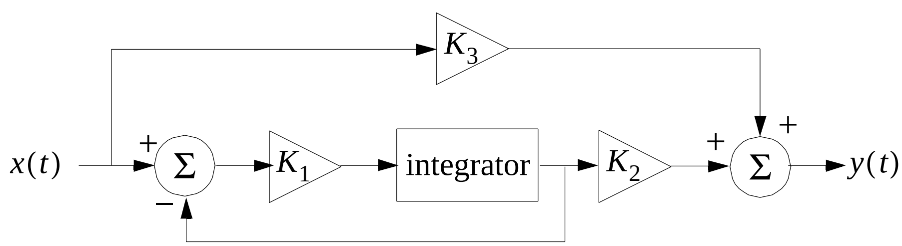
(b) (10 pts.) Without calculating $X(\omega)$ explicitly, use Fourier transform properties to express $X_1(\omega), X_2(\omega),$ and $X_3(\omega)$ in terms of $X(\omega)$. More than one correct answer is possible in some cases.

(c) (15 pts.) Give an explicit expression for $X(\omega)$.

Problem 2 (15 pts.) For each discrete-time system having input $x[n]$ and output $y[n]$, use an "X" to indicate any property that pertains.

System	Linear	Time-Invariant	Memoryless	Causal	Stable
$y[n] = -x[-n]$					
$y[n] = x[n-1] - x[n-4]$					
$y[n] = \sum_{k=0}^{n+2} x[k]$					
$y[n] = (n+1)x[n]$					

Problem 3 (25 pts.) Consider the following continuous-time system having input $x(t)$ and output $y(t)$. The overall system has impulse response $h(t)$ and transfer function $H(s)$.

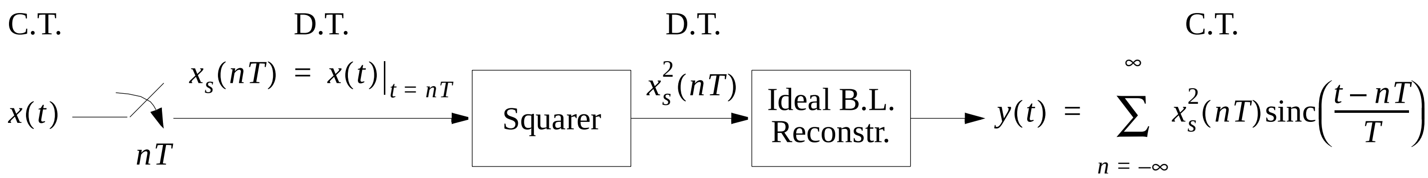


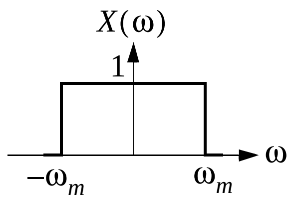
(a) (10 pts.) Find an expression for $H(s)$.

(b) (5 pts.) Find the poles and zeros of $H(s)$. Making reference to $H(s)$, determine for what values of K_1, K_2, K_3 the system is stable.

(c) (10 pts.) Find an expression for $h(t)$. Making reference to $h(t)$ only, determine for what values of K_1, K_2, K_3 the system is stable.

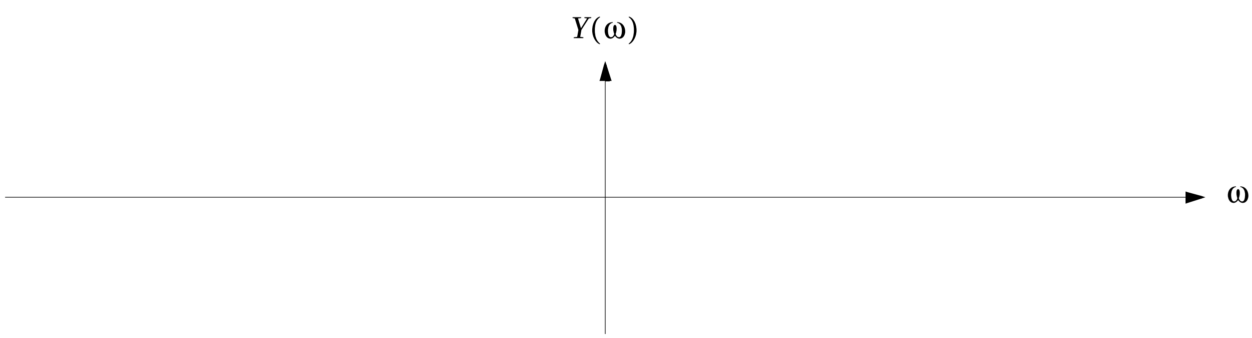
Problem 4 (30 pts.) Consider the following system. $x(t)$ is bandlimited to a bandwidth ω_m.





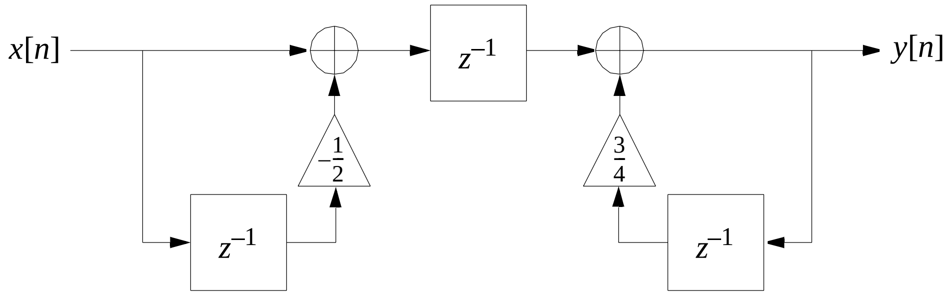
(a) (10 pts.) Define $Z(\omega) \equiv F\{x^2(t)\}$, the Fourier transform of $x^2(t)$. Find an expression for $Y(\omega)$, the Fourier transform of $y(t)$, in terms of $Z(\omega)$.

(b) (10 pts.) Assume $\omega_m = \frac{\pi}{4T}$. Sketch $Y(\omega)$, labeling the horizontal and vertical axes.



(c) (10 pts.) Find $\omega_{m,\,\max}$ such that if $\omega_m < \omega_{m,\,\max}$, then $y(t) = x^2(t)$.

Problem 5 (35 pts.) Consider the following discrete-time system having input $x[n]$ and output $y[n]$:



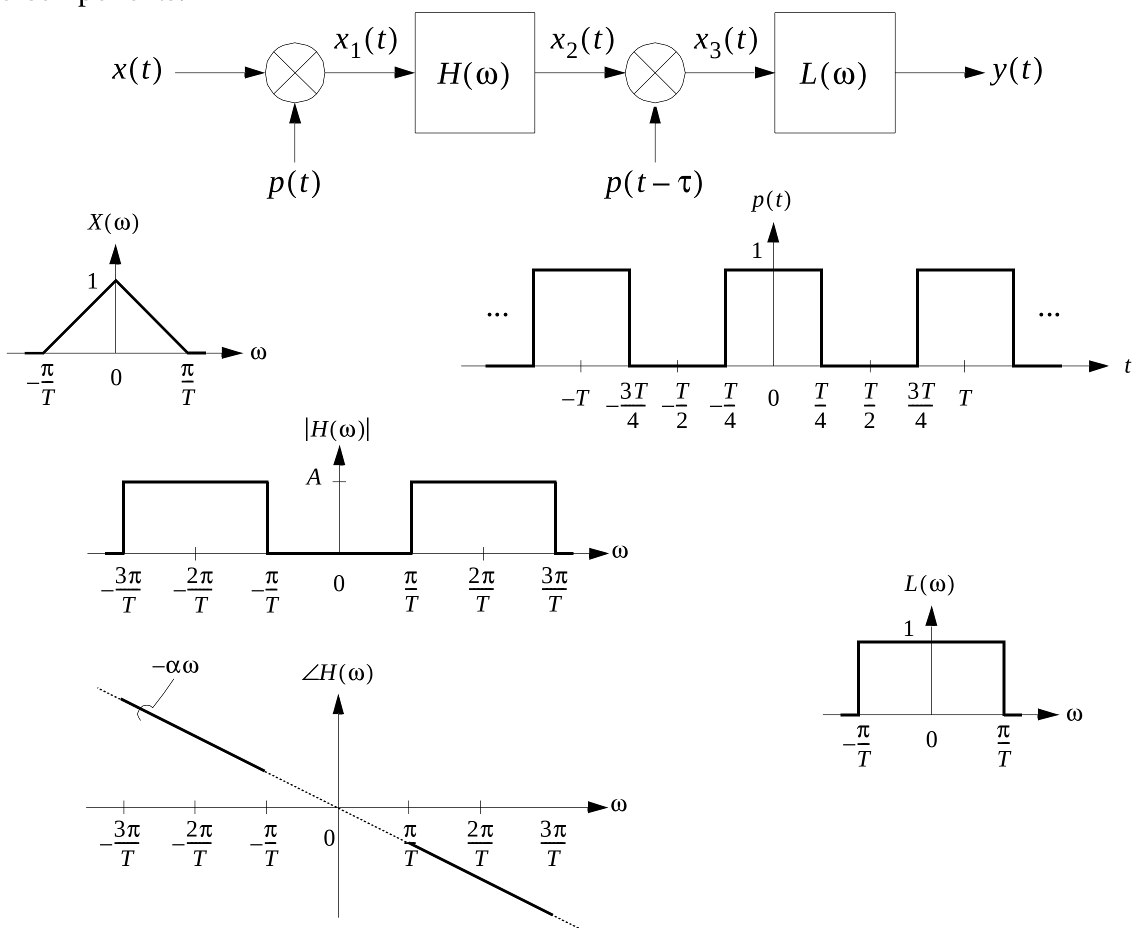
(a) (10 pts.) Find the transfer function $H(z)$.

(b) (10 pts.) Using only two delay elements, draw the block diagram of another system with the same transfer function.

(c) (10 pts.) Find the output $y[n]$ if the input is $x[n] = (1/2)^n u[n]$. Assume zero initial conditions.

(d) (5 pts.) Find the output $y[n]$ if the input is one for all time, $x[n] = 1$.

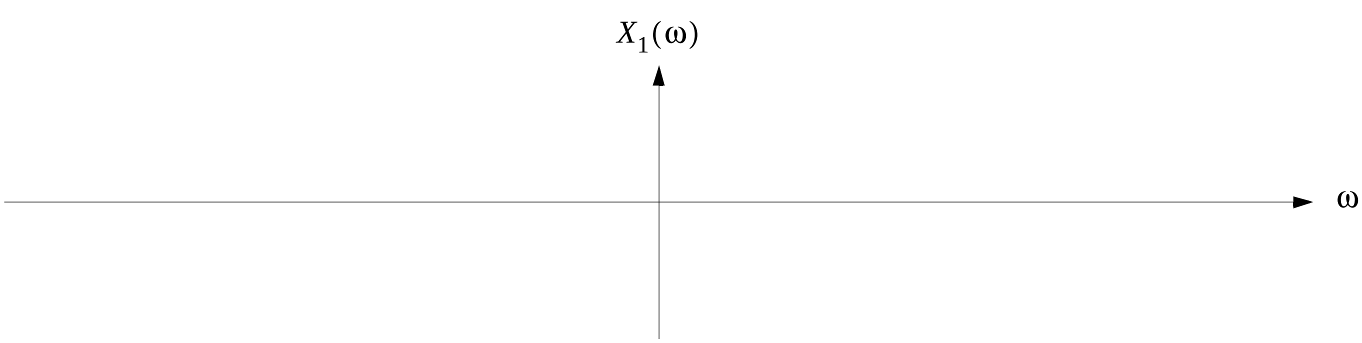
Problem 6 (60 pts.) The design of high-gain, d.c.-coupled amplifiers (amplifiers whose passband includes $\omega = 0$) sometimes presents difficulties because small, slow changes in the quiescent operating points of the transistors can lead to a drift of the output that is indistinguishable from those due to small desired signals. A possible solution is shown here. The input signal $x(t)$ is band-limited as shown. A periodic pulse train $p(t)$ is used to modulate $x(t)$ onto a carrier. A high-gain, bandpass amplifier $H(\omega)$ amplifies the resulting signal. A delayed copy of the pulse train, $p(t-\tau)$, modulates the signal back to baseband, and the lowpass filter $L(\omega)$ removes the undesired pass-band components.



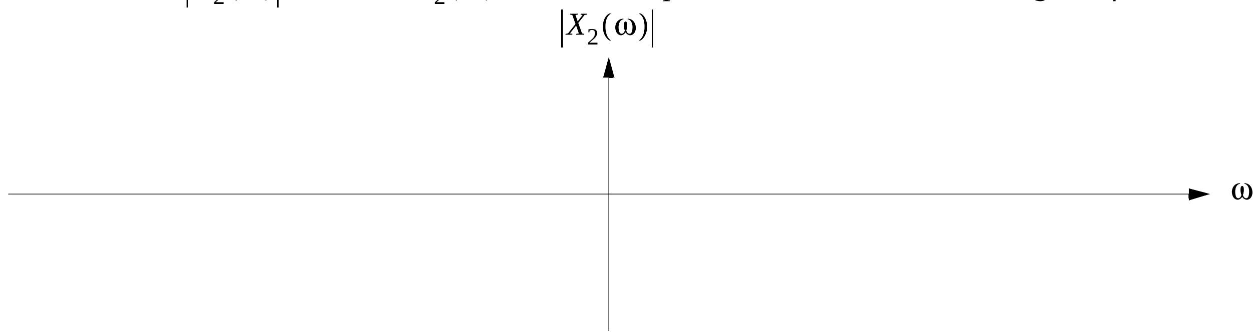
(a) (10 pts.) Write down the Fourier transforms of $p(t)$ and $p(t-\tau)$.

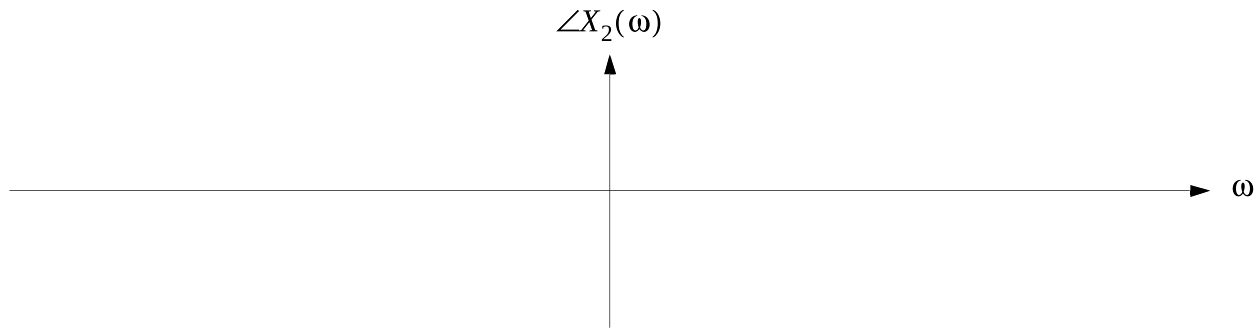
(b) (10 pts.) Write down an expression for $X_1(\omega)$ in terms of $X(\omega)$.

(c) (10 pts.) Sketch $X_1(\omega)$ over the range $\frac{-7\pi}{T} \leq \omega \leq \frac{7\pi}{T}$, labeling the horizontal and vertical axes.



(d) (10 pts.) Find an expression for $X_2(\omega)$ in terms of $X(\omega)$. You may find it helpful to first sketch $|X_2(\omega)|$ and/or $\angle X_2(\omega)$, *but this is optional and no credit will be given for it.*





(e) (10 pts.) Find an expression for $X_3(\omega)$ in terms of $X(\omega)$.

(f) (5 pts.) Find an expression for $Y(\omega)$ in terms of $X(\omega)$.

(g) (5 pts.) Specify a value of τ such that the overall system [with input $x(t)$ and output $y(t)$] achieves the largest possible positive gain. For this choice of τ, find an expression for $y(t)$.