EECS 120
Midterm 1
Wed. Oct. 15, 2014
1610 - 1730 pm

Name:____________________________
SID:______________________

- Closed book. One 8.5x11 inch page one side formula sheet. No calculators.
- There are 4 problems worth 100 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	22	
2	25	
3	26	
4	27	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1} 1 = 45°$
$\tan^{-1}\frac{1}{3} = 18.4°$	$\tan^{-1}\frac{1}{4} = 14°$
$\tan^{-1}\sqrt{3} = 60°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

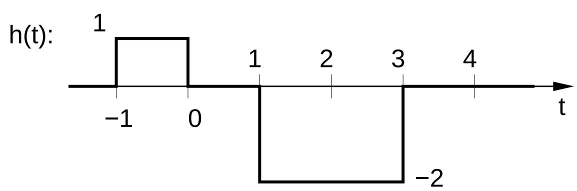
$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB
$1/e \approx 0.37$	$1/e^2 \approx 0.14$
$1/e^3 \approx 0.05$	$\sqrt{10} \approx 3.16$
$\pi \approx 3.14$	$2\pi \approx 6.28$
$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 LTI Properties (22 pts)

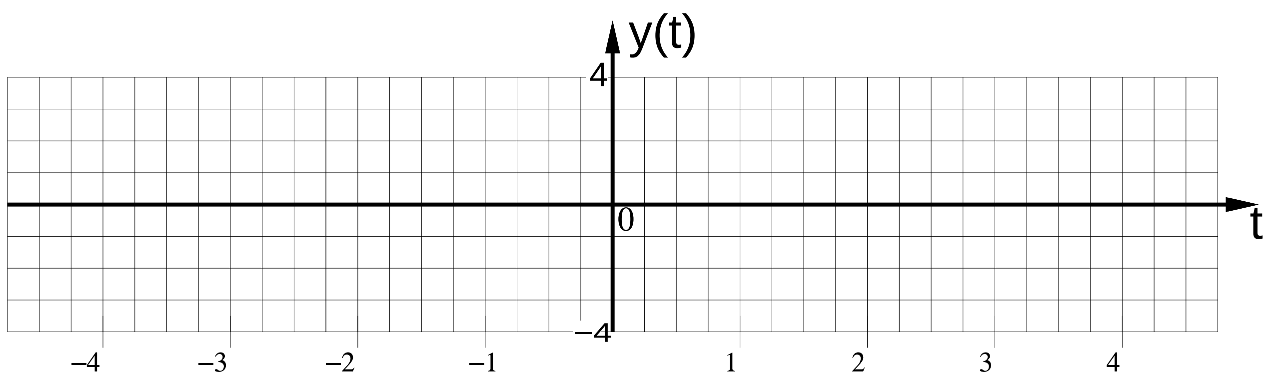
[16 pts] a. Classify the following systems, with input $x(t)$ and output $y(t)$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).

System	Causal	Linear	Time-invariant	BIBO
a. $y(t) = x(t) \cdot \Sigma_{n=-\infty}^{\infty} \delta(t-2n)$				
b. $y(t) = x(t) * \Sigma_{n=0}^{\infty} \delta(t-2n)$				
c. $y(t) = x(t) - \frac{1}{2}\frac{dx(t+1)}{dt}$				
d. $y(t) = \int_{-1}^{1} x(\tau)x(t-\tau)d\tau$				

[6 pts] e. An LTI system has impulse response $h(t)$ as shown below:

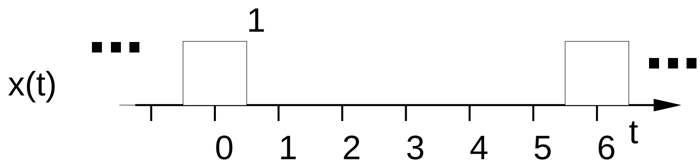


Given input $x(t) = u(t+1)$. Sketch the output $y(t)$ on the grid below, noting key times and amplitudes.



Problem 2 Fourier Series (25 pts)

You are given a periodic function $x(t)$ as shown, where the shape is a rectangular pulse of height 1 and width 1, centered at $t = 0$:



Note that $x(t)$ can be represented by a Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t},$$

where $a_k = \frac{\sin k\pi/6}{k\pi}$.

[1 pts] a. What is the fundamental frequency $\omega_o =$ __________

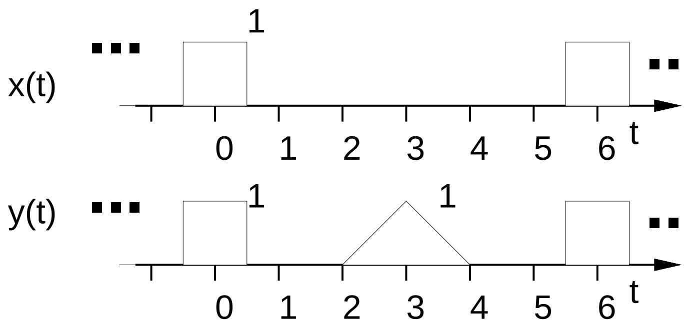
[2 pts] b. What is the total time average power in $x(t)$? __________

[5 pts] c. What is the percentage of the total power in $x(t)$ which is not at DC or the fundamental frequency?

percent = __________

Problem 2, continued.

Given a new signal $y(t)$ as shown:



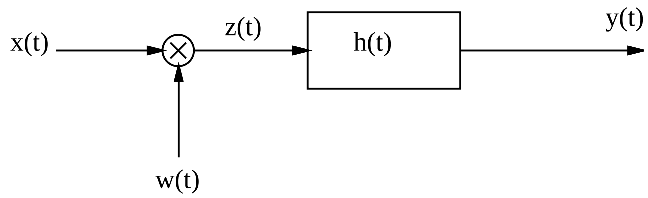
Periodic function $y(t)$ can be represented by a Fourier Series:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_o t}$$

[12 pts] d. Find $b_k =$ ________________________

[5 pts] e. If $y(t) = x(t) * h(t)$, find $h(t) =$: ________________________

Problem 3. Fourier Transform (26 pts)
For each part below, consider the following system:



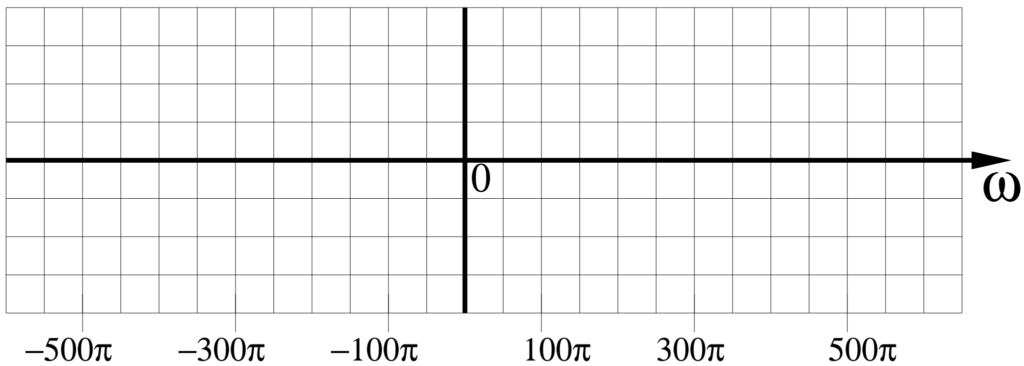
Where $x(t) = \cos(400\pi t) + \Pi(\frac{t}{4T_o})$, $\quad w(t) = \frac{1}{2T_o}\Pi(\frac{t}{2T_o})$, $\quad h(t) = \Sigma_{n=-\infty}^{\infty}\delta(t - \frac{n}{50})$ with $T_o = 1/100$ sec.

(Recall that $\Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$.)

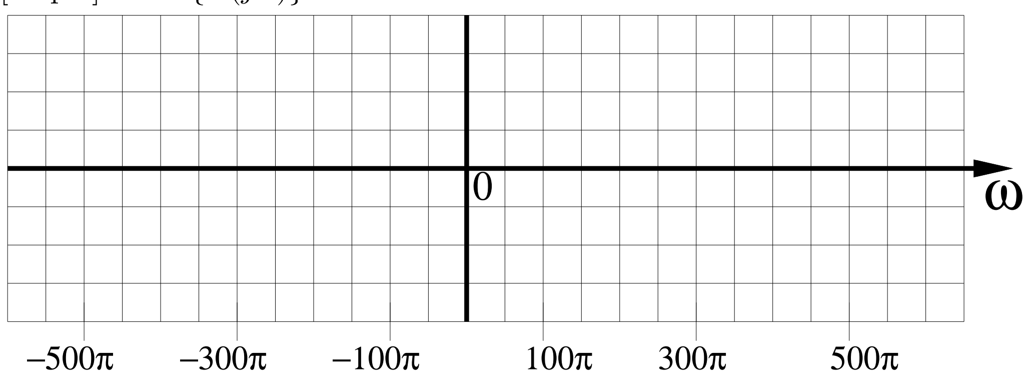
On the next page, sketch $Re\{X(j\omega)\}, Re\{Z(j\omega)\}, Re\{Y(j\omega)\}$ labelling height/area, center frequencies, and key zero crossings for $-500\pi \leq \omega \leq 500\pi$:

Problem 3, continued.

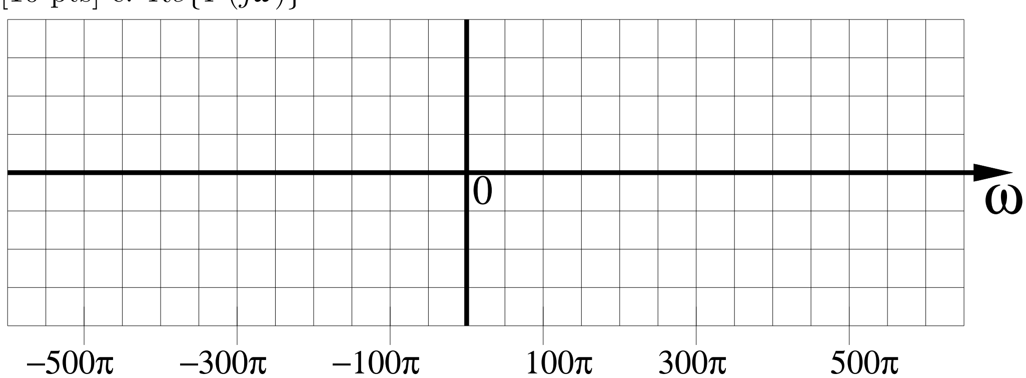
[6 pts] a. $Re\{X(j\omega)\}$



[10 pts] b. $Re\{Z(j\omega)\}$



[10 pts] c. $Re\{Y(j\omega)\}$



Problem 4. DTFT (27 points)

A causal LTI system with input $x[n]$ and output $y[n]$ is described by the transfer function:

$$H(e^{j\omega}) = \frac{j\sin\omega}{\cos\omega}$$

[5 pts] a. Find the difference equation relating $y[n]$ and $x[n]$, corresponding to $H(e^{j\omega})$:

$y[n] =$ ____________________________________

[7 pts] b. Find the impulse response $h[n]$, that is, the time response of the system to input $x[n] = \delta[n]$.

$h[n] =$_________

[10 pts] c. If $x[n] = 2\cos(\frac{\pi n}{3})$ find $y[n]$. $y[n] =$ _________

Problem 4, continued.

[5 pts] d. Let $z[n] = \cos[\frac{\pi n}{4}] \cos[\frac{\pi n}{2}]$. Find the DTFT of $z[n]$.

$Z(e^{j\omega}) =$ ________________________