**EE 120: Signals and Systems** MIDTERM 2
Department of EECS 26 October 2010
UC Berkeley

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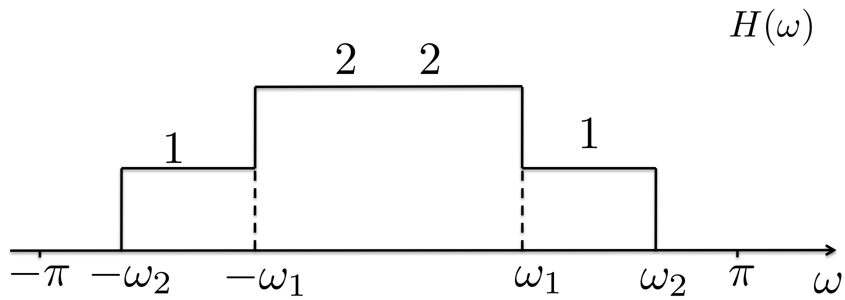
LAST Name ______________________ FIRST Name ______________________

Discussion Time ____________________

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except **two** double-sided 8.5" $\times$11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

**MT2.1 (45 Points)** Consider a discrete-time LTI system H whose frequency response $H(\omega)$ is shown below for $-\pi \le \omega \le \pi$.

For every part of this problem, let $\omega_1 = \pi/3$ rad/sec, and $\omega_2 = 2\pi/3$ rad/sec.




(a) (10 Points) Determine a reasonably simple, closed-form expression for $h(n)$, the impulse response of the system.

(b) (i) (5 Points) Without explicitly carrying out the infinite sum, evaluate $\sum_{n=-\infty}^{+\infty} h(n)$.

(ii) (5 Points) Without explicitly carrying out the infinite sum, evaluate $\sum_{n=-\infty}^{+\infty} |h(n)|^2$.

(c) (10 Points) Let $x$ denote the input to the LTI system $\mathsf{H}$. For each of the following choices of $x(n)$, determine a reasonably simply closed-form expression for the corresponding output $y(n)$.

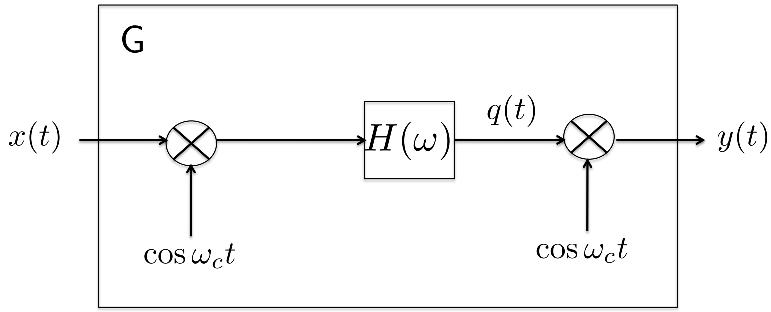
(i) (5 Points) $x(n) = \cos\left(\frac{\pi n}{6}\right) + \cos\left(\frac{7\pi n}{12}\right) + \sin\left(\frac{3\pi n}{4}\right), \quad \forall n \in \mathbb{Z}$.

(ii) (5 Points) $x(n) = \sum_{\ell=-\infty}^{+\infty} \delta(n - 4\ell), \quad \forall n \in \mathbb{Z}.$

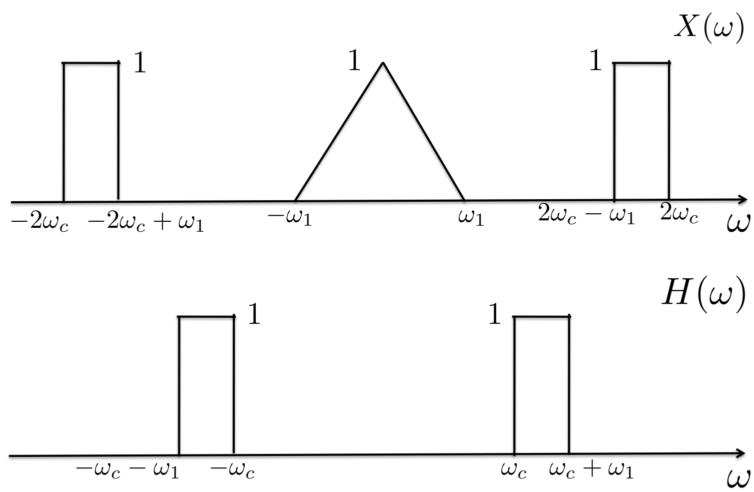
(d) (15 Points) In practice, non-causal filters are more difficult to implement and use than causal filters. Suppose we make a causal approximation $\widehat{h}(n)$ to the LTI system by forming $\widehat{h}(n) = h(n)\,u(n)$, where $u(n)$ is the discrete-time unit step. Let the approximation error be $e(n) = h(n) - \widehat{h}(n)$. Evaluate (i.e., find a numerical value for) the energy of the approximation error, which can be expressed by

$$\mathcal{E} = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |H(\omega) - \widehat{H}(\omega)|^2 d\omega.$$

**MT2.2 (25 Points)** A continuous-time system G consists of three processing blocks: modulation, followed by LTI filtering, followed by another modulation, as the figure below indicates.



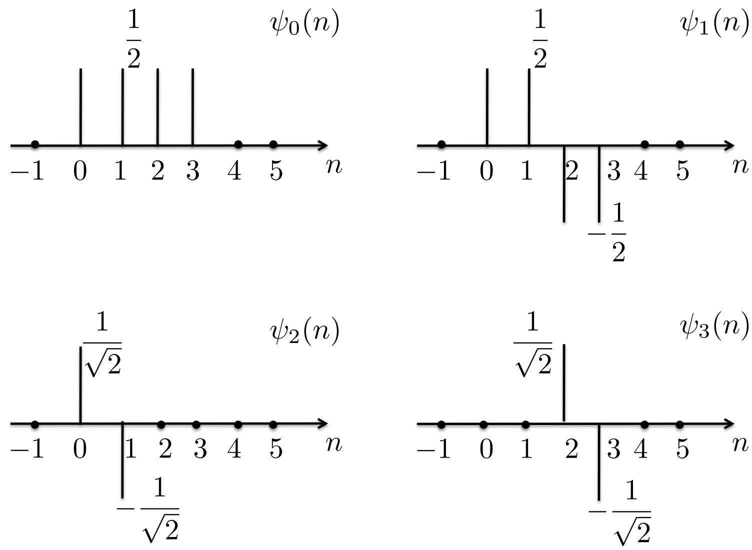

The spectrum $X(\omega)$ of the input signal $x$, as well as the frequency response $H(\omega)$ of the LTI filter, are depicted below. Assume that $\omega_c$ is sufficiently large; in particular, $\omega_c > 2\omega_1$.




(a) (15 Points) Provide well-labeled plots of $Q(\omega)$ and $Y(\omega)$, the spectra of $q$ and $y$, respectively. (Feel free to use the blank space at the top of the next page to show your work for this part).

(b) (10 Points) Determine a reasonably simple closed-form expression for $h(t)$, the impulse response of the filter.

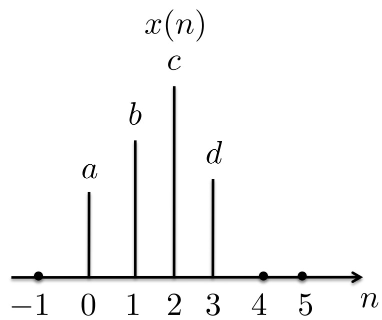
**MT2.3 (40 Points)** Let G denote the set of all real-valued discrete-time signals having a region of support $[0, 3]$; every signal in G is zero outside the interval $[0, 3]$. Consider the signals $\{\psi_k\}_{k=0}^{3}$ as shown below. Note that every $\psi_k(n) = 0$ for $n \notin \{0, 1, 2, 3\}$.




(a) (6 Points) Show that G is a subspace of $\ell^2(\mathbb{Z})$, the vector-space of all finite-energy discrete-time signals. Explain whether G is itself a vector space.

(b) (6 Points) Show that $\{\psi_k\}_{k=0}^{3}$ is an orthonormal basis for G.

(c) (8 Points) Given a signal $x$ as shown below, determine the coefficients $X_k$ in the expansion $x(n) = \sum_{k=0}^{3} X_k \, \psi_k(n)$.



(d) (5 Points) Let $\mathsf{H} = \text{span}\{\psi_0, \psi_1\}$ be a subspace of $\mathsf{G}$ corresponding to the span of the first two basis elements $\psi_0$ and $\psi_1$. We want to approximate $x$ by $\widehat{x} \in \mathsf{H}$ where $\widehat{x} = \sum_{k=0}^{1} \alpha_k \psi_k$.

What choice of coefficients $\{\alpha_k\}_{k=0}^{1}$ minimize the approximation error energy, given by $\mathcal{E} = \sum_{n=0}^{3} |e(n)|^2$, where $e = x - \widehat{x}$ denotes the error. You may express your answers in terms of the coefficients $X_k$ of part (c).

(e) (10 Points) Let the signal $x$ of part (c) be defined by $a = 2$, $b = 0$, $c = 1$, and $d = -1$. That is, $x(0) = 2$, $x(1) = 0$, $x(2) = 1$, $x(3) = -1$, and $x(n) = 0$ for all other values of $n$.

Suppose you are given a choice of *exactly two* of the basis functions $\psi_k$ with which to construct an approximate $\widehat{x}$ to the signal $x$ (e.g., $\widehat{x} = \beta_1 \psi_1 + \beta_3 \psi_3$). Which two (among $\{\psi_k\}_{k=0}^{3}$) would you choose? Explain your choice of the two $\psi_k$'s and determine the energy of the approximation error signal $e = x - \widehat{x}$ for your particular estimate $\widehat{x}$.

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| Problem | Points | Your Score |
|---|---|---|
| Name | 10 | |
| 1 | 45 | |
| 2 | 25 | |
| 3 | 35 | |
| **Total** | **115** | |