EE 120: Signals and Systems
Department of EECS
UC Berkeley

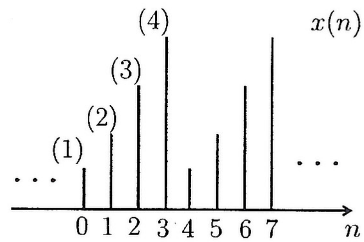
MIDTERM 1
22 September 2010

LAST Name ______________________ FIRST Name ______________________

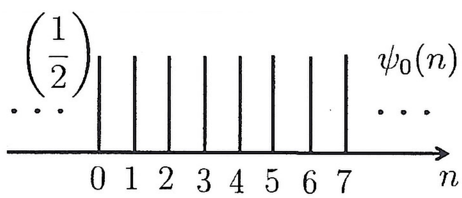
Discussion Time ______________________

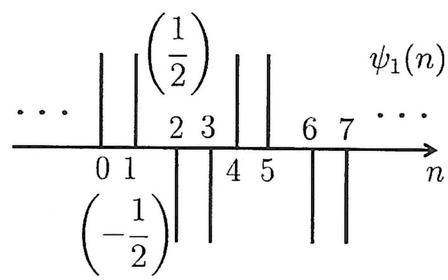
- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

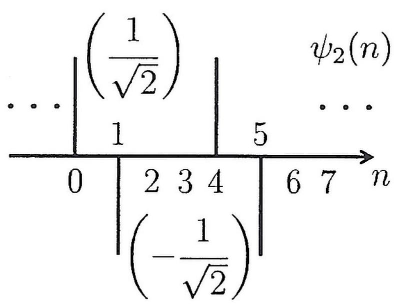
MT1.1 (40 Points) Consider a discrete-time signal x having period $p = 4$.

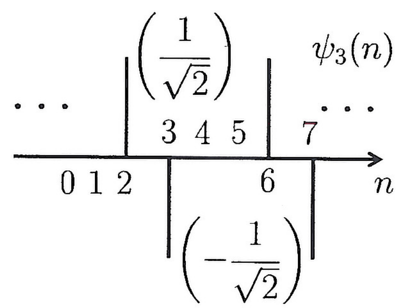


We want to express x as a linear combination of signals $\{\psi_k\}_{k=0}^{3}$, each of period 4.









(a) Show that $\langle \psi_k, \psi_\ell \rangle = \delta(k - \ell)$, for all $k, \ell \in \{0, 1, 2, 3\}$.

(b) Determine all the coefficients X_k in the decomposition

$$x(n) = \sum_{k=0}^{3} X_k \, \psi_k(n), \quad \text{for all } n.$$

(c) Show that, for our particular choice of signals $\{\psi_k\}_{k=0}^{3}$,

$$\sum_{k=0}^{3} |x(n)|^2 = \sum_{k=0}^{3} |X_k|^2,$$

and evaluate $\sum_{k=0}^{3} |X_k|^2$.

(d) Evaluate $\sum_{k=0}^{3} X_k$.

MT1.2 (40 Points) The frequency response of a causal discrete-time LTI filter H is

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \frac{1}{1 + \frac{1}{2}e^{-i\omega}}.$$

(a) Determine the linear, constant-coefficient difference equation that characterizes the input-output behavior of the filter.

(b) Determine $h(n)$, the impulse response values of the filter.

(c) Provide a well-labeled plot of the filter's magnitude response $|H(\omega)|$ and phase response $\angle H(\omega)$ for $|\omega| < \pi$. You must explain how you arrive at the plot.

(d) We apply the following periodic discrete-time signal to the filter:

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{\ell=-\infty}^{+\infty} 2\,\delta(n - 1 - 8\ell).$$

(i) Determine a reasonably simple expression for the discrete-time Fourier series[1] coefficients X_k of the input signal, where

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{k=\langle 8 \rangle} X_k\, e^{i2\pi kn/8}.$$

(ii) Express, in terms of X_k and the filter's frequency response $H(\omega)$, the discrete-time Fourier series coefficients Y_k of the corresponding output signal y. Also, find Y_4, in particular.

[1] The complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p: $x(n) = \sum_{k=\langle p \rangle} X_k\, e^{ik\omega_0 n} \quad \longleftrightarrow \quad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n)\, e^{-ik\omega_0 n}$, where $p = \frac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\sum_{k=\langle p \rangle}$ may denote $\sum_{k=0}^{p-1}$ or $\sum_{k=1}^{p}$.

MT1.3 (25 Points) The input-output behavior of a discrete-time system F is described below:

$$y(n) = \sum_{k=-\infty}^{-2n} 3x(k).$$

(a) Select the strongest correct assertion from the choices below. Explain your reasoning succinctly, but clearly and convincingly.

(i) F must be a linear system.

(ii) F can be a linear system.

(iii) F cannot be a linear system.

(b) Select the strongest correct assertion from the choices below. Explain your reasoning succinctly, but clearly and convincingly.

(i) F must be a time-invariant system.

(ii) F can be a time-invariant system.

(iii) F cannot be a time-invariant system.

(c) Select the strongest correct assertion from the choices below. Explain your reasoning succinctly, but clearly and convincingly.

(i) F must be a causal system.

(ii) F can be a causal system.

(iii) F cannot be a causal system.

(d) Select the strongest correct assertion from the choices below. Explain your reasoning succinctly, but clearly and convincingly.

(i) F must be a BIBO stable system.

(ii) F can be a BIBO stable system.

(iii) F cannot be a BIBO stable system.

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Problem	Points	Your Score
Name	10	
1	40	
2	40	
3	25	
Total	**115**	