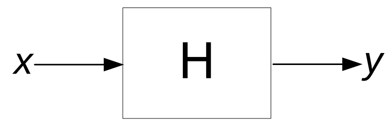
EE 120: Signals and Systems MIDTERM 1
Department of EECS 23 September 2009
UC BERKELEY

LAST Name ____________________ FIRST Name ____________________

Discussion Time ____________________

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (35 Points) Consider a continuous-time LTI system H as shown below:



The impulse response of the system is characterized as follows:

$$\forall t \in \mathbb{R}, \quad h(t) = \cos(\omega_0 t)\, e^{-at}\, u(t),$$

where $a > 0$, u is the continuous-time unit-step, and $\omega_0 \neq 0$.

(a) Provide a well-labeled plot of the impulse response h. To receive credit, you must explain (briefly) how you determine the plot.

(b) Show that the frequency response of the system is

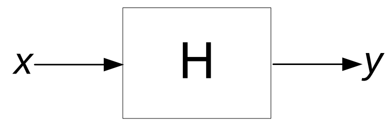
$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \frac{a + i\omega}{(a + i\omega)^2 + \omega_0^2}.$$

(c) For this part, assume $0 < a < 1 \ll \omega_0$.

(i) Provide a well-labeled plot of the magnitude response $|H(\omega)|$. You must explain how you determine the plot.

(ii) Determine a reasonable approximation to the output signal values $y(t)$, for all t, if the input signal values are described by $x(t) = 1 + \cos(\omega_0 t)$. Explain your work succinctly, but clearly and convincingly.

MT1.2 (35 Points) Consider a continuous-time LTI system H as shown below:



(a) The impulse response of the system is known to be real-valued: that is, $h(t) \in \mathbb{R}$, for all t. If the input signal is $x(t) = \cos(\omega_0 t)$, for all t, then show that the output signal is described by $y(t) = |H(\omega_0)| \cos\big(\omega_0 t + \angle H(\omega_0)\big)$, for all t, where $H(\omega)$ is the frequency response value at frequency ω.

(b) Now assume that the impulse response of the system is described by $h(t) = e^{-at}\, u(t)$, for all t, where $a > 0$ and u is the continuous-time unit-step. Determine the output signal values $y(t)$, for all t, if the input signal values are described by $x(t) = \cos(at)$. Provide a well-labeled plot of the output signal y, if $a = \pi/4$.

MT1.3 (35 Points) Consider the finite-duration discrete-time signal x defined by

$$\forall n \in \mathbb{Z}, \quad x(n) = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine a reasonably simple expression for, and provide a well-labeled plot of, $X(\omega)$, the DTFT values of the signal x. Explain your work.

(b) Construct a periodic extension of x as follows: $\widetilde{x}(n) = \displaystyle\sum_{\ell=-\infty}^{+\infty} x(n - \ell p)$, for all n, where p is a positive integer no smaller than 3. Determine $\widetilde{X}_k$, the DFS coefficients of the periodic signal $\widetilde{x}$, where $\widetilde{x}(n) = \displaystyle\sum_{k=\langle p \rangle} \widetilde{X}_k \, e^{ik\omega_0 n}$, and $\omega_0 = 2\pi/p$. How do the coefficients $\widetilde{X}_k$ relate to $X(\omega)$?

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Problem	Points	Your Score
Name	10	
1	35	
2	35	
3	35	
Total	**115**	