EE 120: Signals and Systems MIDTERM 1
Department of EECS 1 October 2008
UC BERKELEY

LAST Name ______________________ FIRST Name ______________________

Discussion Time ______________________

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

MT1.1 (30 Points) The following discrete-time systems F, G, H should be treated mutually independently; properties that hold for one system *cannot* be assumed to hold for the others.

For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) A discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y described by

$$y(n) = \left(\frac{1}{2}\right)^n, \quad \forall n,$$

in response to the input signal x characterized by

$$x(n) = \left(\frac{1}{2}\right)^n u(n), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be BIBO stable.

(ii) The system could be BIBO stable, but does not have to be.

(iii) The system cannot be BIBO stable.

(b) A *time-invariant* discrete-time system $\mathsf{G} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y described by

$$y(n) = 2^n u(-n-1), \forall n\,,$$

in response to the input signal x characterized by

$$x(n) = \left(\frac{1}{2}\right)^n u(n), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

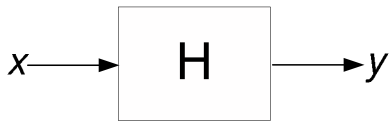
(c) A discrete-time system $\mathsf{H} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces an output signal y described by $y = \mathsf{Re}(x)$, in response to every input signal x. Select the strongest true assertion from the list below.

(i) The system must be linear.

(ii) The system could be linear, but does not have to be.

(iii) The system cannot be linear.

MT1.2 (45 Points) Consider a discrete-time FIR filter $\mathsf{H} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ having impulse response h and frequency response H.



Suppose the impulse response h is a finite-length rectangular pulse, described by

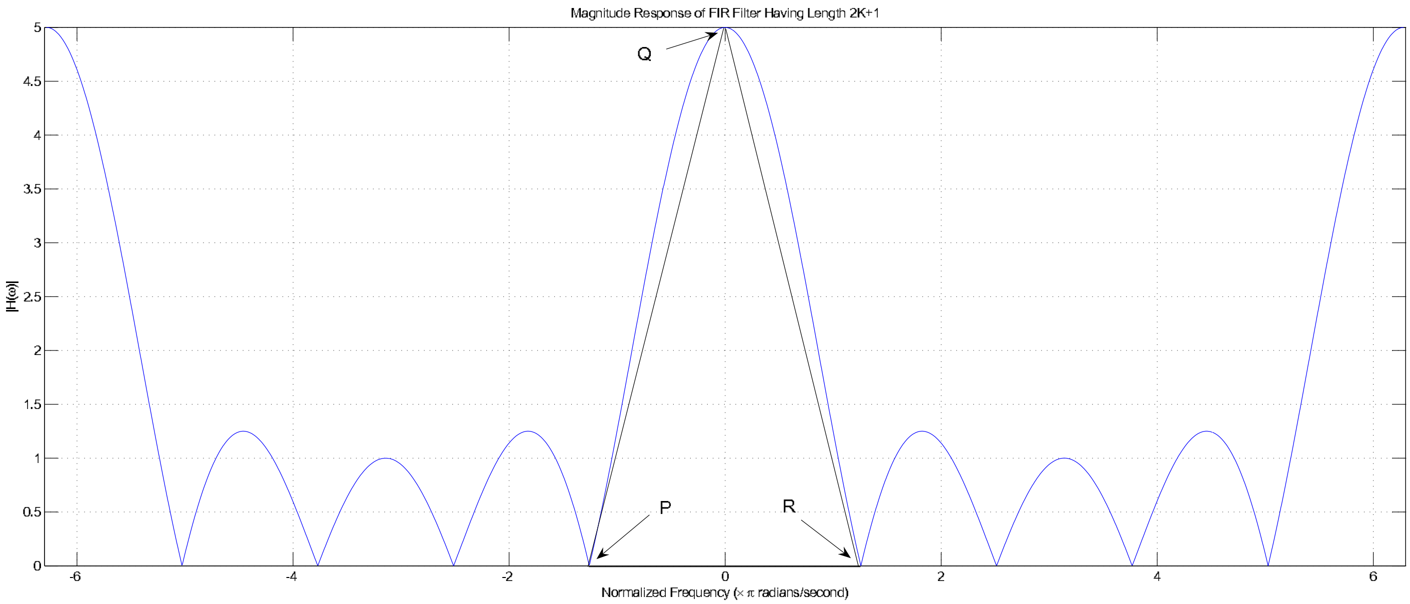
$$h(n) = \begin{cases} A & |n| \leq K \\ 0 & |n| > K \,, \end{cases}$$

where $A > 0$ and $K \in \{1, 2, 3, \ldots\}$.

(a) Determine (in terms of A, K, or both) a reasonably simple expression for the frequency response $H(\omega)$.

(b) Determine (in terms of A, K, or both) a simple expression for $\displaystyle\int_{\langle 2\pi \rangle} H(\omega)\, d\omega$, the area under the filter's frequency response curve.

(c) The figure below depicts $|H(\omega)|$, the magnitude of the filter's frequency response, for a particular value of A and a particular value of K. The plot shows the region $-2\pi < \omega < 2\pi$.



(i) Determine (in terms of A, K, or both) the frequencies at which $|H(\omega)| = 0$. Then compute the particular numerical values of A and K that produced the magnitude response plot above. Be sure to label on the plot the values of all the frequencies at which $H(\omega) = 0$.

(ii) Determine (in terms of A, K, or both) the area of the triangle PQR inscribed within the main lobe of $|H(\omega)|$, and compare your answer with what you found in part (b) above.

(iii) Suppose the input signal x is defined as the periodic extension of a finite-duration signal x_0 as follows:

$$x_0(n) = -\delta(n{+}1){+}3\,\delta(n){-}\delta(n{-}1) \qquad \text{and} \qquad x(n) = \sum_{\ell=-\infty}^{+\infty} x_0\Big(n{-}(2K{+}1)\ell\Big)$$

Determine the response of the filter to the input signal x. If you're confident in the numerical values you found in part (c)(i), you may express your answer here numerically. Otherwise, you may express your answer in terms of A, K, or both.

MT1.3 (30 Points) Parts (a) and (b) of this problem are mutually independent and may be tackled in either order.

(a) A full-wave rectified sine wave x is described by $x(t) = |\sin(\pi t)|, \quad \forall t.$

(i) Determine the complex exponential Fourier series coefficients of x.
Note: You may find the following integral useful:

$$\int_a^b e^{\lambda t}\, dt = \frac{e^{b\lambda} - e^{a\lambda}}{\lambda}.$$

(ii) Use your result in Part (a)(i) to prove the identity

$$\sum_{k=-\infty}^{+\infty} \frac{(-1)^k}{1-4k^2} = \frac{\pi}{2}.$$

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(b) A continuous-time signal x_{CT}, characterized by $x_{\mathsf{CT}}(t) = \cos(\omega_0 t)$, $\forall t$, is sampled every T seconds to produce a discrete-time signal x_{DT} described by $x_{\mathsf{DT}}(n) = x_{\mathsf{CT}}(nT)$, $\forall n$.

Determine a condition on the sampling period T (in terms of ω_0) to guarantee that x_{DT} is periodic.

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

LAST Name ____________________ FIRST Name ____________________

Discussion Time ____________________

Problem Name	Points 10	Your Score
1	30	
2	45	
3	30	
Total	**115**	