EECS 120 **Fall 2004**

Midterm 2

• The exam is for one hour and 50 minutes.

• The maximum score is 100 points. The maximum score for each part of each problem is indicated.

• The exam is closed-book and closed-notes. Calculators, computing, and communication devices are NOT permitted.

• Four double-sided sheets of notes are allowed. These should be legible to normal eyesight, i.e. the lettering should not be excessively small.

• No form of collaboration between students is allowed.

1. *(8 points)* State whether the following are true or false. In each case, give a brief explanation. A correct answer with a correct explanation gets 4 points. A correct answer without a correct explanation gets 1 points. A wrong answer gets 0 points.

   (a) If two signals $x(t)$ and $y(t)$ have the same *unilateral* Laplace transform, then the signals are identical.

   (b) Let the signal $x_1(t)$ have Laplace transform $X_1(s)$ with region of convergence R_1, and let the signal $x_2(t)$ have Laplace transform $X_2(s)$ with region of convergence R_2. Assume that $R_1 \cap R_2$ is not the empty set. Then the signal $y(t) = x_1(t) * x_2(t)$, which is the convolution of $x_1(t)$ with $x_2(t)$, has Laplace transform $Y(s) = X_1(s)X_2(s)$ with region of convergence equal to $R_1 \cap R_2$.

2. *(6 + 2 + 6 + 2 + 4 + 5 points)* Consider the impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} [\delta(t - 3n\Delta) + \delta(t - 3n\Delta - \Delta) - \delta(t - 3n\Delta - 2\Delta)] \ .$$

   Note carefully that every third impulse has sign opposite to that of the preceding two impulses.

   Let $x(t)$ be a signal. We assume that $x(t)$ is band limited to $(-\omega_M, \omega_M)$, i.e.

$$X(j\omega) = 0 \ , \quad \text{if } |\omega| \geq \omega_M \ ,$$

   where $X(j\omega)$ denotes the Fourier transform of $x(t)$.

   Let

$$x_p(t) = x(t)p(t) \ .$$

   Let $H(j\omega)$ be given by

$$H(j\omega) = \begin{cases} A & \text{if } -\omega_f < \omega < \omega_f \\ 0 & \text{otherwise .} \end{cases}$$

Thus the filter with frequency response $H(j\omega)$ is a low pass filter which passes only frequencies in the range $-\omega_f < \omega < \omega_f$, with amplification A in the pass band.

Let $y(t)$ denote the output of this low pass filter when the input is $x_p(t)$.

(a) Let $P(j\omega)$ denote the Fourier transform of $p(t)$. Determine $P(j\omega)$.

(b) Choose some $X(j\omega)$ that is non-zero over $(-\omega_M, \omega_M)$ and has linear phase over $(-\omega_M, \omega_M)$, with the phase being non-zero except at $\omega = 0$. Sketch the magnitude and phase plots for the $X(j\omega)$ that you picked. Make sure to label all the important frequency, magnitude, and phase coordinates.

(c) Let $X_p(j\omega)$ denote the Fourier transform of $x_p(t)$. Sketch the magnitude and phase plots for $X_p(j\omega)$ corresponding to the $X(j\omega)$ that you picked, assuming the appropriate no-aliasing condition. Make sure to label all the important frequency, magnitude, and phase coordinates.

(d) What is the appropriate no-aliasing condition in the preceding part of the problem ?

(e) Find the conditions on ω_M, Δ, ω_f, and A which ensure that $y(t) = x(t)$ for all $x(t)$ that are band limited to $(-\omega_M, \omega_M)$.

(f) Is there a way to recover $x(t)$ from $x_p(t)$ even when the no-aliasing condition of part (d) does not hold ? Explain your answer.

3. *(12 points)*

Let $x(t)$ be a low pass signal that is bandlimited to $(-\omega_M, \omega_M)$, i.e. if $X(j\omega)$ denotes the Fourier transform of $x(t)$ then we have

$$X(j\omega) = 0 \text{ if } |\omega| \geq \omega_M .$$

Let $y(t) = A_c x(t) \cos(\omega_c t)$ denote the DSB-SC signal resulting from amplitude modulation of $x(t)$ onto the carrier $A_c \cos(\omega_c t)$. Assume that $\omega_c >> \omega_M$.

Let $p(t)$ denote the periodic signal with period $T_c = \frac{2\pi}{\omega_c}$ given by

$$p(t) = \begin{cases} 1 & \text{if } |t - nT_c| \leq \frac{T_c}{4} \text{ for some } n \in \mathbf{Z} \\ 0 & \text{otherwise} \end{cases}$$

Let $z(t) = y(t)p(t)$. The signal $z(t)$ is passed through an ideal low pass filter $H(j\omega)$ with cutoff frequency ω_M, i.e.

$$H(j\omega) = \begin{cases} 1 & \text{if } |\omega| \leq \omega_M \\ 0 & \text{otherwise} \end{cases} .$$

Let the output of this filter be denoted $w(t)$.

Determine $w(t)$.

4. *(2 + 6 + 6 points)*

Let $x[n]$ be a discrete time signal whose discrete time Fourier transform (DTFT) $X(e^{j\omega})$ is bandlimited to $(-\frac{\pi}{10}, \frac{\pi}{10})$, i.e.

$$X(e^{j\omega}) = 0 \text{ if } \mid \omega \mid \leq \pi \text{ and } \mid \omega \mid \geq \tfrac{\pi}{10} .$$

Recall that

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

is a periodic function of ω with period 2π, which is why the extra condition that $\mid \omega \mid \leq \pi$ is needed in the notion of "bandlimited".

Let $z[n]$ be created from $x[n]$ by dropping all terms for n an integer multiple of 4, i.e.

$$z[n] = \begin{cases} 0 & \text{if } n \text{ is a multiple of 4} \\ x[n] & \text{otherwise} . \end{cases}$$

You can think of $z[n]$ as $x[n] - y[n]$, where $y[n]$ is the discrete time sampling of $x[n]$ with period 4, if you like.

Let $w[n]$ denote the up-sampled version of $z[n]$ with up-sampling factor 3, i.e. to get $w[n]$ we first create

$$v[n] = \begin{cases} z[\frac{n}{3}] & \text{if } n \text{ is a multiple of 3} \\ 0 & \text{otherwise} \end{cases} ,$$

and then pass $v[n]$ through an ideal discrete time low pass filter that is bandlimited to $(-\frac{\pi}{3}, \frac{\pi}{3})$.

For the following, pick some $X(e^{j\omega})$ that is periodic with period 2π, non-zero over $(-\frac{\pi}{10}, \frac{\pi}{10})$, and has linear phase over $(-\frac{\pi}{10}, \frac{\pi}{10})$ which is non-zero except at $\omega = 0$.

(a) Sketch the magnitude and phase plots for the $X(e^{j\omega})$ you picked. Make sure to label all the important frequency, magnitude, and phase coordinates.

(b) Let $Z(e^{j\omega})$ denote the DTFT of $z[n]$. Sketch the magnitude and phase plots of $Z(e^{j\omega})$ corresponding to the $X(e^{j\omega})$ that you picked. Make sure to label all the important frequency, magnitude, and phase coordinates.

(c) Let $W(e^{j\omega})$ denote the DTFT of $w[n]$. Sketch the magnitude and phase plots of $W(e^{j\omega})$ corresponding to the $X(e^{j\omega})$ that you picked. Make sure to label all the important frequency, magnitude, and phase coordinates.

5. *(10 points)*

Let $x[n]$ be a discrete time signal. Assume that its discrete time Fourier transform (DTFT) $X(e^{j\omega})$, which is a periodic function with period 2π, is band limited to $(-\omega_M, \omega_M)$, i.e. assume that

$$X(e^{j\omega}) = 0 , \quad \text{if } \omega_M <\mid \omega \mid \leq \pi .$$

The extra condition that $\mid \omega \mid \leq \pi$ is needed in the notion of "bandlimited" because $X(e^{j\omega})$, is periodic with period 2π. We assume, of course, that $\omega_M < \pi$.

Assume that $x[n] \neq 0$ for all n.

Let $y[n]$ be a discrete time signal with DTFT $Y(e^{j\omega})$. Assume that this is also bandlimited to $(-\omega_M, \omega_M)$.

Let $a > 1$ be a real number such that $a\omega_M < \pi$, and let

$$z[n] = \cos(a\ \omega_M n) + y[n]\ .$$

Let $v[n] = x[n]z[n]$. The signal $v[n]$ is periodically sampled with period N. Here $N \geq 1$ is some integer.

For what values of a and N is it possible to recover *both* $x[n]$ and $y[n]$ from the samples $v[nN], n \in \mathbf{Z}$?

6. *(2 + 2 + 9 points)*

A causal LTI system has system function

$$H(s) = \frac{s-4}{s^2+5s+4}\ .$$

(a) Determine the region of convergence of the system function.

(b) Is the system stable ? Why ?

(c) Determine the output of the system when the input is

$$x(t) = e^{-2|t|}\ .$$

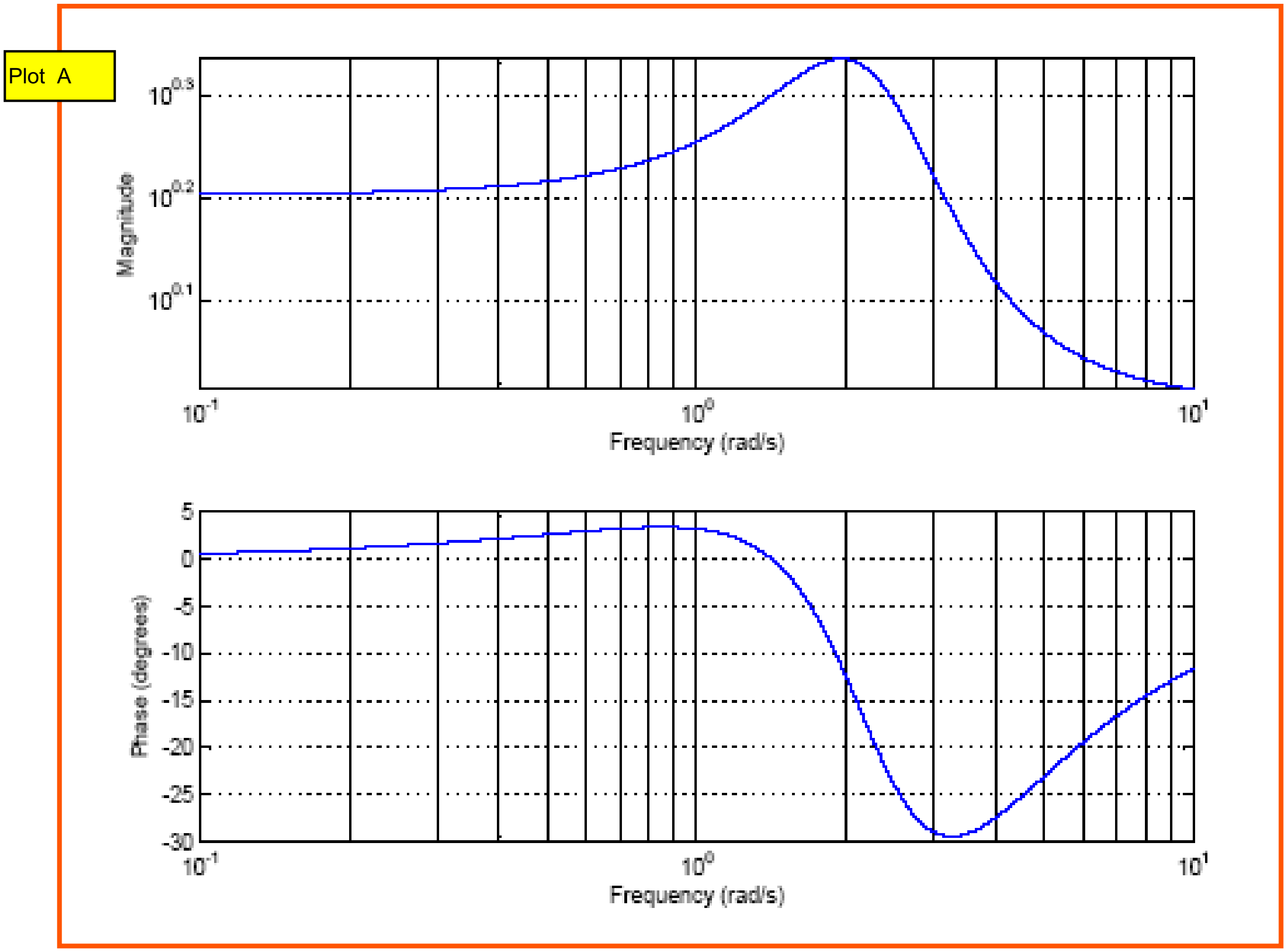
7. *(18 points)*

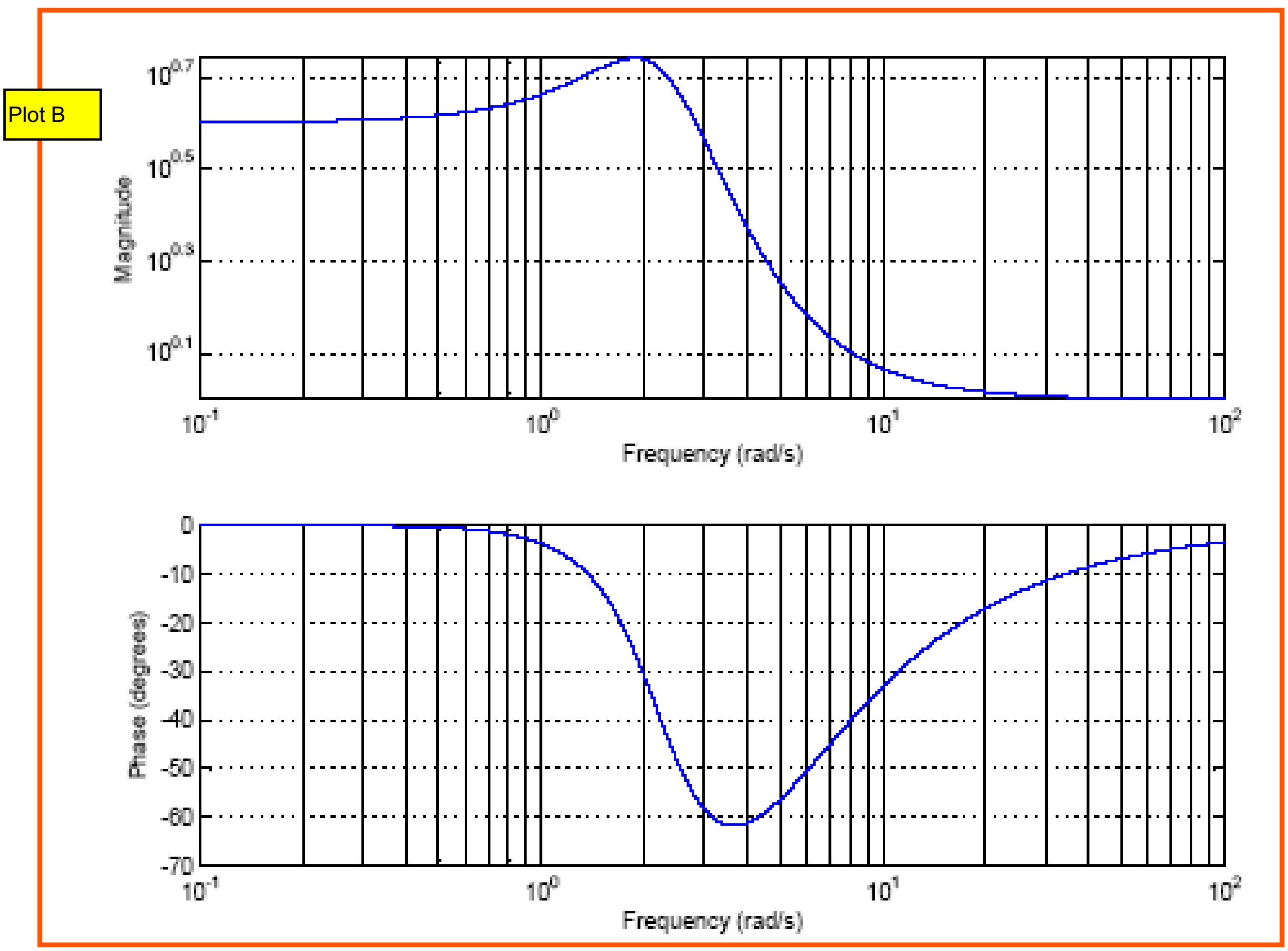
Each of the plots A, B, C, and D on the next two pages is a pair, giving $\mid H(j\omega) \mid$ as a function of $\omega > 0$ in radians on a log-log plot and $\angle H(j\omega)$ in degrees as a function of $\omega > 0$ in radians on a linear-log plot. These plots correspond, in some unknown order, to the frequency responses of the causal stable LTI systems with the following system functions (the finite poles and zeros of these system functions are also explicitly given for convenience).

Number	System function	poles	zeros
1.	$\frac{1}{s^2+2s+5}$	$-1+j2,\ -1-j2$	none
2.	$\frac{s^2+4s+8}{s^2+2s+5}$	$-1+j2,\ -1-j2$	$-2+j2,\ -2-j2$
3.	$\frac{s^2-8s+20}{s^2+2s+5}$	$-1+j2,\ -1-j2$	$4+j2,\ 4-j2$
4.	$\frac{s^2+8s+20}{s^2+2s+5}$	$-1+j2,\ -1-j2$	$-4+j2,\ -4-j2$

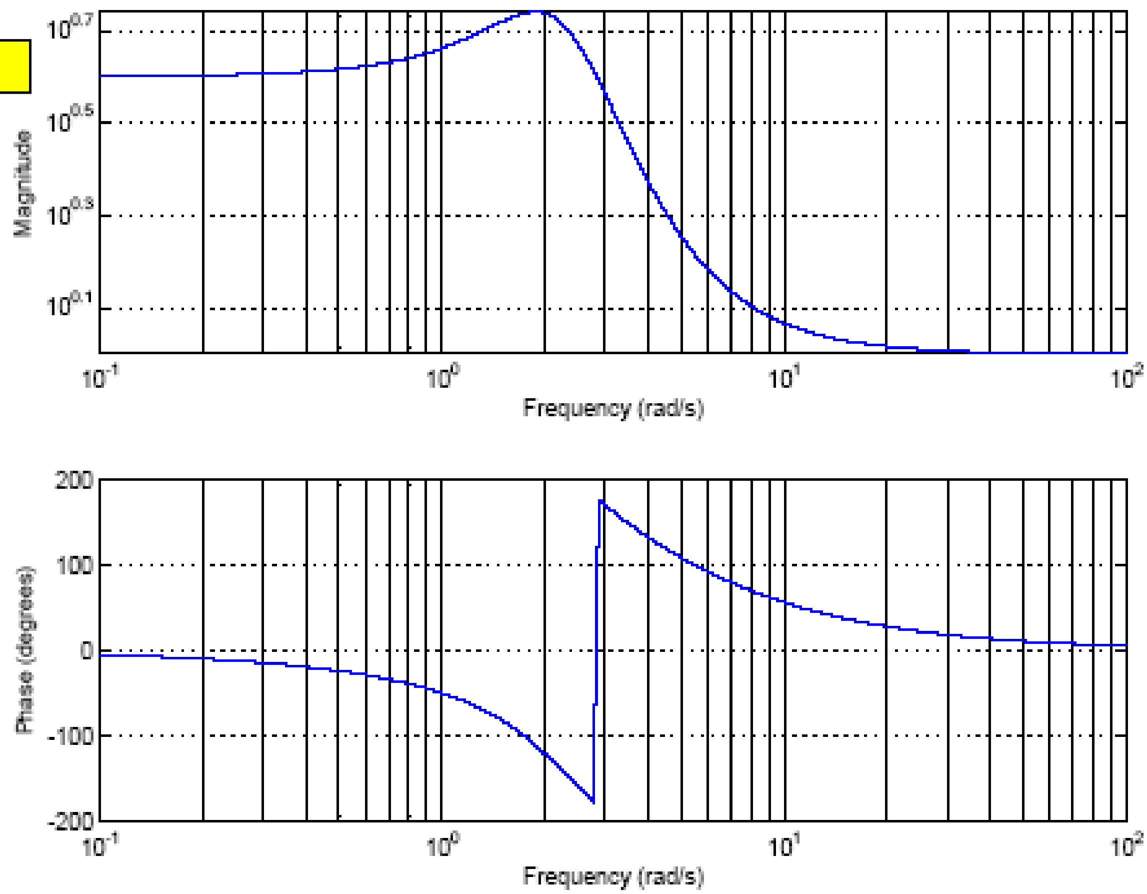
Match up the system functions with the corresponding plots. Every correct answer gets 3 points, and every wrong answer gets -1 points. Also, explain your reasoning for why you matched the system functions to the plots the way you did. 6 points are allocated to evaluate your reasoning.

Note : Even though negative points will be given for wrong answers, the lowest score possible on this problem as a whole will be zero points.


Plot A
Magnitude
$10^{0.3}$
$10^{0.2}$
$10^{0.1}$
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)
Phase (degrees)
5
0
-5
-10
-15
-20
-25
-30
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)

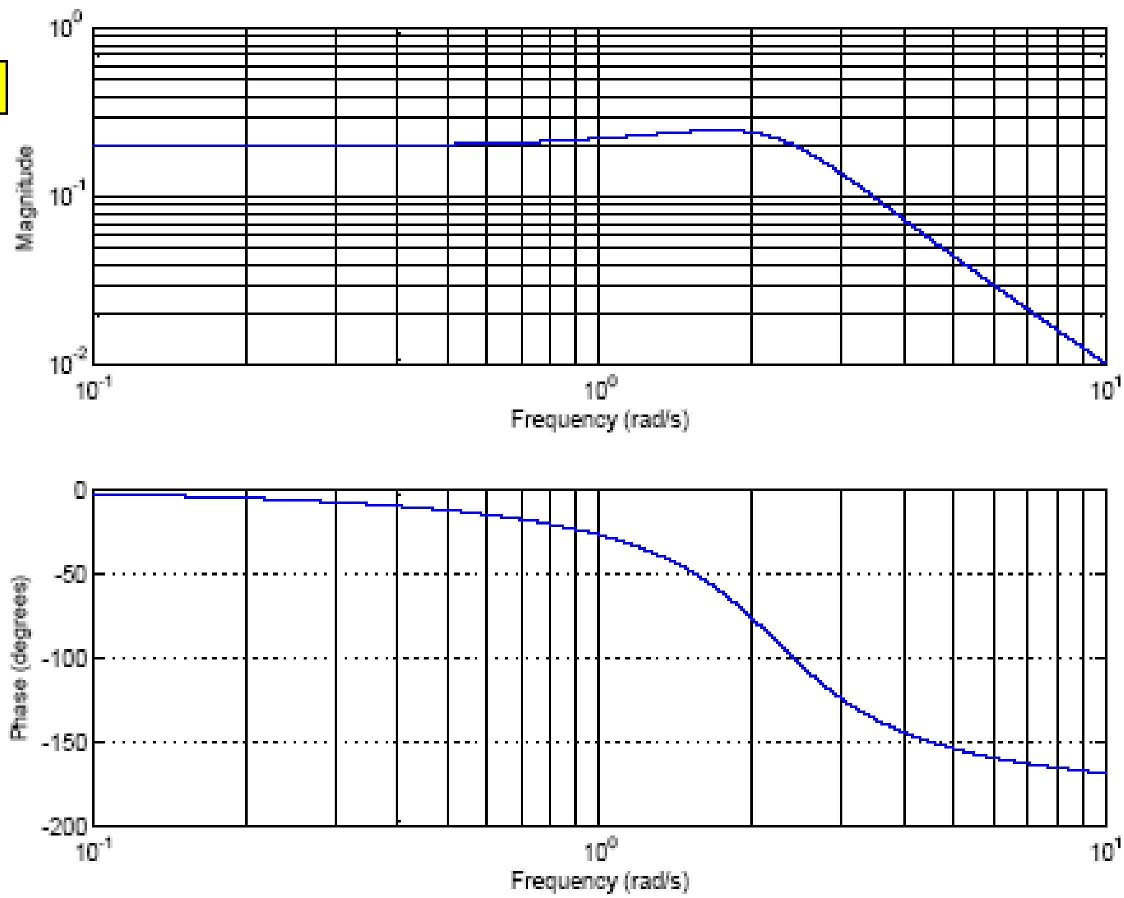

Plot B
Magnitude
$10^{0.7}$
$10^{0.5}$
$10^{0.3}$
$10^{0.1}$
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)
Phase (degrees)
0
-10
-20
-30
-40
-50
-60
-70
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

Plot C



$10^{0.7}$
$10^{0.5}$
$10^{0.3}$
$10^{0.1}$
Magnitude
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)
200
100
0
-100
-200
Phase (degrees)
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

Plot D



10^{0}
10^{-1}
10^{-2}
Magnitude
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)
0
-50
-100
-150
-200
Phase (degrees)
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)