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MIDTERM EXAM - EECS 118

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PRINT YOUR NAME: ____

Last First

Do "all work" on this exam.

Make your methods clear.

There are 3 problems.

Problem 1) : (30 points)
A plane wave of the form $\bar{\mathbf{E}} = \bar{\mathbf{A}}e^{i\omega t - i\bar{\mathbf{k}}\cdot\bar{\mathbf{r}}}$ travels through a material with a frequency dependent refractive index given by $n(\omega) = n_0 + n_1(\omega - \omega_0)$

a)What is the magnitude of $\bar{\mathbf{k}}$ in terms of the frequency ω , the speed of light c , n_0, n_1 and ω_0 ?

$$k(\omega) = (\omega/c) \times n(\omega) = \omega/c\,(n_0 + n_1(\omega - \omega_0))$$

b) A second plane wave of the form $\bar{\mathbf{E}} = \bar{\mathbf{A}}e^{i\omega_1 t - i\bar{\mathbf{k}}_1\cdot\bar{\mathbf{r}}}$ is now simultaneously present. What is the magnitude of $\bar{\mathbf{k}}_1$?

$$k(\omega_1) = \omega_1/c\; n(\omega_1) = \omega_1/c\,(n_0 + n_1(\omega_1 - \omega_0))$$

c) What are the phase speeds of the two waves in a) and b) ?

phase speeds are $\omega_1/k(\omega_1)$ and $\omega/k(\omega)$

$$\omega_1/k(\omega_1) = c/n(\omega_1) \quad ; \quad \omega/k(\omega) = c/n(\omega)$$

d) If $\omega_1 - \omega \ll \omega$ what is the group speed in terms of the same parameters as in a) and what is it that travels at this speed ?

v_g = group speed $\lim_{\Delta\omega \to 0} \frac{\Delta\omega}{\Delta k}$ $\quad \Delta\omega = \omega - \omega_1$, $\quad \Delta k = k(\omega) - k(\omega_1)$

$$v_g = \frac{\omega - \omega_1}{\omega/c\,(n_0 + n_1(\omega - \omega_0)) - \omega_1/c\,(n_0 + n_1(\omega_1 - \omega_0))}$$

$$= \frac{\omega - \omega_1}{\left(\frac{\omega - \omega_1}{c}\right)n_0 + n_1/c\,(\omega(\omega - \omega_0) - \omega_1(\omega_1 - \omega_0))}$$

$$= \frac{\omega - \omega_1}{\left(\frac{\omega - \omega_1}{c}\right)n_0 + n_1/c\,((\omega - \omega_1)(\omega + \omega_1) - \omega_0(\omega - \omega_1))}$$

$$= c\,\frac{\cancel{\omega - \omega_1}}{\cancel{\omega - \omega_1}}\;\frac{1}{(n_0 + n_1(\omega + \omega_1 - \omega_0))} \xrightarrow{\text{as } \omega \to \omega_1} \frac{c}{(n_0 + n_1(2\omega - \omega_0))}$$

Note

$$\frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{1}{\frac{n_0}{c} + \frac{n_1}{c}(2\omega - \omega_0)}$$

Consider $\bar{A}$ real

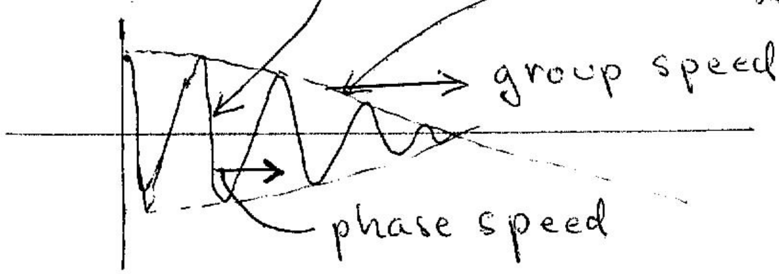
Then the wave is

$$\mathrm{Re}\,\bar{E} = \bar{A}\left(\cos(\omega t - kz) + \cos(\omega_1 t - k(\omega_1) z)\right)$$

$$= \cancel{\bar{A}\, 2 \left[\cos\left[\frac{\omega - \omega_1}{2} t + \frac{k(\omega) - k(\omega_1)}{2} z\right] \times \sin\left[\frac{\omega + \omega_1}{2} t + \frac{k(\omega) + k(\omega_1)}{2} z\right]\right]}$$

$$= \frac{\bar{A}}{2}\left(\left[e^{i\frac{\omega + \omega_1}{2} t - i\frac{k(\omega) + k(\omega_1)}{2} z} \right] \left\{ \frac{e^{i\frac{\omega - \omega_1}{2} t - i\frac{k(\omega) \mp k(\omega)}{2} z}}{2} + c.c. \right\} + c.c. \right)$$

$$= \bar{A} \cos\left(\frac{\omega + \omega_1}{2} t - \frac{k(\omega) + k(\omega_1)}{2} z\right) \times \cos\left(\frac{\omega - \omega_1}{2} t - \frac{k(\omega) - k(\omega_1)}{2} z\right)$$



Problem 2) : (30 points)

A simple model of a receiver is a detector (assume p-i-n) in parallel with a resistor of value R and an ideal amplifier having a constant gain G over the bandwidth of interest. It is assumed that the capacitance can be neglected.

a) If it is assumed that R is the input resistance of the amplifier and the noise figure of the amplifier is F_n, what is the thermal plus amplifier noise observed at the output of the amplifier?

$$\frac{4kT}{R}\Delta f + \frac{4kT}{R}(F_n - 1)\Delta f = \frac{4kT}{R}F_n \Delta f$$

b) If the optical power is P_o , what is the shot noise observed at the output of the amplifier in terms of P_o and the responsivity R_s of the detector?

$$2e I_{Avg}\Delta f = 2e R_s P_o \Delta f$$

c) What is the signal to noise ratio at the output of the amplifier?

$$\frac{R_s P_o}{\left(2e R_s P_o \Delta f + \frac{4kT}{R} F_n \Delta f\right)^{1/2}} = K$$

d) Show that the maximum signal to noise ratio is obtained in the shot noise limit.

Solve c for P_o

$$(K)^2 \left(2e R_s P_o \Delta f + \frac{4kT}{R} F_n \Delta f\right) = R_s^2 P_o^2$$

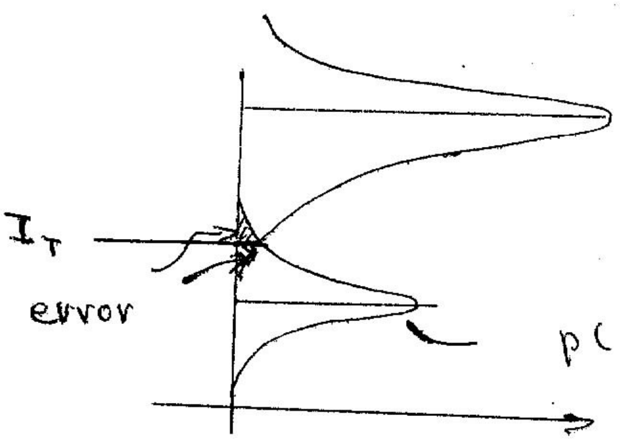
$$P_o^2 - K^2 \frac{2e P_o \Delta f}{R_s} - \frac{4kT F_n \Delta f}{R_s^2 R} K^2 = 0$$

$$P_o = \frac{K^2 e \Delta f}{R_s} + \left(\left(\frac{K^2 e \Delta f}{R_s}\right)^2 + \frac{4kT F_n \Delta f K^2}{R_s^2 R}\right)^{1/2}$$

Minimum when thermal noise term is 0

Then $P_o = \dfrac{2K^2 e \Delta f}{R_s}$ which is the shot noise limit

d) For a simple amplitude shift keyed digital format for which no power is transmitted for the zero bit and Gaussian noise statistics are assumed, what is the bit error rate in terms of the error function?



$$p(I, I_1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-(I-I_1)^2/2\sigma_1^2}$$

$$p(I, I_0) = \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-(I-I_0)^2/2\sigma_0^2}$$

$$P.E. = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\left\{\frac{1}{\sigma_0}\int_{I_T}^{\infty} e^{-(I-I_0)^2/2\sigma_0^2}\, dI + \frac{1}{\sigma_1}\int_{-\infty}^{I_T} e^{-(I-I_1)^2/2\sigma_1^2}\, dI\right\}\right.$$

Minimize P.E. $\frac{\partial P.E.}{\partial I_T} = 0$

Approximate $\sigma_0 \simeq \sigma_1$ (note $\frac{\partial}{\partial I_T}\int_{I_T}^{\infty} f(I)\, dI = -f(I_T)$

$\int_{-\infty}^{I_T} f'(I)\, dI = f'(I_T)$) Thus $e^{-(I_T-I_0)^2/2\sigma_0^2} = e^{-(I_T-I_1)^2/2\sigma_1^2}$

Thus $\frac{I_T - I_0}{\sigma_0} \simeq \pm\left(\frac{I_T - I_1}{\sigma_1}\right)$; Thus $I_T \simeq \frac{\sigma_1 I_0 + \sigma_0 I_1}{(\sigma_1+\sigma_0)}$

Change first integration varible to $\frac{I-I_0}{\sigma_0}$ and second to $\frac{I-I_1}{\sigma_1}$

$\frac{I_T - I_0}{\sigma_0} = \frac{\cancel{\sigma_0}(I_1 - I_0)}{\cancel{\sigma_0}(\sigma_1+\sigma_0)}$ and $\frac{I_T - I_1}{\sigma_1} = \frac{I_0 - I_1}{\sigma_1+\sigma_0}$; In the

2'nd integral $\int_{-\infty}^{I_0-I_1/\sigma_0+\sigma_1} (\quad)\, d\frac{I - I_1}{\sigma_1} = \int_{(I_1-I_0)/\sigma_0+\sigma_1}^{\infty} (\quad)\, d\frac{I_1 - I}{\sigma_1}$ and

both integrals are equal, Thus B.E.R. $= \frac{1}{2}\operatorname{erfc}(x)$

and $x = \frac{(I_1 - I_0)}{\sqrt{2}(\sigma_0+\sigma_1)} = \frac{(R_s P_0 - 0)}{\sqrt{2}\left((4kT/R\, F_n \Delta f)^{1/2} + (4kT/R\, F_n\Delta f + 2eR_sP_0\Delta f)^{1/2}\right)}$ ASK Format

Problem 3) (30 points) A diffraction limited laser with a (minimum) beam width equal to w at z = 0 has a message imposed upon it (it is modulated). It is transmitted a length L to a lens of diameter D which focusses it onto a detector having a diameter d.

a) If the transmitted power from the laser is P_{opt} what fraction of P_{opt} strikes the detector? Give your answer in terms of the parameters of the problem.

if w is Gaussian beam radius

Θ = cone angle = $\dfrac{\lambda}{2\pi w_o}$

Power/Area = $\dfrac{P_{opt}}{\pi\left(\dfrac{L\lambda}{2\pi w_o}\right)^2}$

b) What should d be to collect all the power striking the lens?

$\left(\dfrac{f}{D}\lambda\right) = d$

c) If an L.E.D. emitting in a cone angle of 1000 $\dfrac{\lambda}{w}$ with the same power is used instead of the laser how much is the received power reduced? (For the L.E. D. the spectral width is of course much larger which further imposes limits which we neglect here)

Cone angle is 1000 times larger. This comes in inversely as the square thus 10^{-6} of a)

continued

Lens focusses to a spot diameter approximately $(F.N.)\lambda = \left(\dfrac{f}{D}\right)\lambda$

$\therefore$ the power collected is

$$\underbrace{\frac{P_{opt}}{\pi\left(\dfrac{L\lambda}{2\pi w_o}\right)^2}}_{W/m^2 \text{ at lens}} \times \underbrace{\pi\left(\frac{D}{2}\right)^2}_{\text{Area of Lens}} \times \underbrace{\frac{1}{\pi\left(\lambda\dfrac{f}{D}\times\dfrac{1}{2}\right)^2}}_{W/m^2 \text{ at focal spot}} \times \underbrace{\pi\left(\frac{d}{2}\right)^2}_{\text{Area of Detector}}$$