Problem 1 of 4: Answer each question briefly and clearly. (5 pts each, 40 total)

1.1 Assume a diffused resistor with a fixed thickness. What happens to the Sheet Resistance (Rs) if the doping level doubles (assuming that there is only a single dopant and that mobility does not change)?

- ☐ … stays the same
- ☐ … doubles
- ☒ … is reduced by 50%
- ☐ … none of the above.

$\sigma = q\mu n \qquad n = N_D$

if N_D doubles σ doubles

$\frac{1}{\sigma}$ gets reduced by 50%

1.2 Find the "built-in potential" ϕ_B of a junction that has $N_A=10^{16}/cm^3$ and $N_D=10^{17}/cm^3$.

$\varphi_n = 60mV \times 7 = 420mV$

$\varphi_p = -60mV \times 6 = -360mV$

$\varphi_B = \varphi_n - \varphi_p = +780mV$

(using the 60mV rule)

1.3 Place check marks where appropriate to indicate the correct region of operation, assuming that $V_{Tn} = -V_{Tp} = 1V$.

Transistor	V_{GS}	V_{DS}	V_{BS}	Off	Triode	Saturation
NMOS	2	2	0		~~X~~	X
NMOS	2	0.5	0		X	
PMOS	2	2	0	X		
PMOS	-2	-0.5	0		X	

1.4 Choose the most appropriate answer.
A negatively charged "depletion" region in doped Si is characterized by…

- ☐ … an abundance of electrons and negative ions
- ☐ … too many electrons, too few holes
- ☒ … a severe imbalance between negative ions and holes
- ☐ … the complete depletion of charge density

(In fact, we only have fixed, negative ions)

1.5 We have a 2-terminal device that accumulates charge as a function of voltage according to the equation: $Q = 5V^2 + 3V + 2$, where Q is in pico Cb and V is in volts. Find the small signal capacitance of this device in pF at 2V.

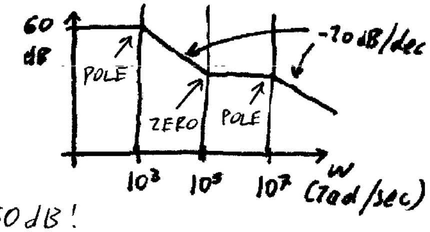
$C = \frac{\partial Q}{\partial V} = (2 \cdot 5 \cdot V + 3) \frac{\text{pico Cb}}{V}$

for $V = 2V \quad \frac{\partial Q}{\partial V} = 23 \frac{\text{picoCb}}{V} = 23pF$

1.6 Write the expression of a transfer function that corresponds to the Bode plot drawn.

$$\frac{1000\,(1 + jw/10^5)}{\left(1 + \frac{jw}{10^3}\right)\left(1 + \frac{jw}{10^7}\right)}$$

$20\log_{10}(1000) = 60dB!$



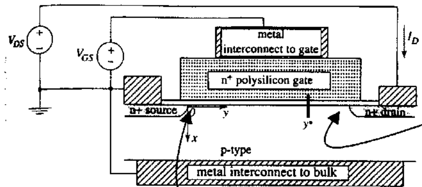
1.7 The value of a voltage source is $5 + 0.02\cos(10^6 t + 45^o)$ Volts. Please complete the list below:

$V_S =$ 5 large signal

$v_s =$ $0.02\cos(10^6 t + 45^o)$ small signal

$v_S =$ $5 + 0.02\cos(10^6 t + 45^o)$ total signal

1.8 For the n-channel MOS transistor shown below, please consider the two ends of the channel (near the source and near the drain) and indicate whether or not each must be inverted so that this device is in saturation.

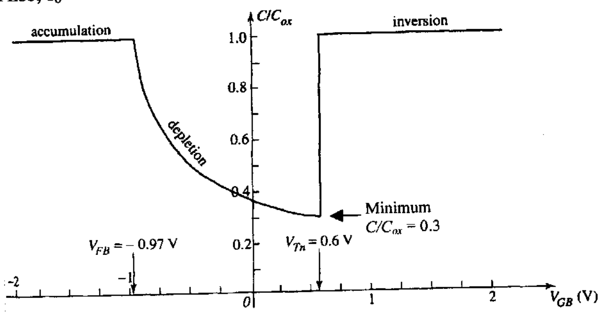


Drain end:
- ☐ This end is inverted
- ☒ This end is NOT inverted

Source end:
- ☒ This end is inverted
- ☐ This end is NOT inverted

Problem 2 of 4: Answer each question briefly and clearly. (20 points)

The C/V plot below was taken by "scanning" the gate-to-bulk voltage of an MOS structure. We know (from other measurements) that $\phi_{n+} = 0.55V$ and $C_{ox} = 2.3 fF/\mu m^2$. Also, $\varepsilon_o = 8.85\ 10^{-14}$ F/cm, $\varepsilon_{ox} = 3.9\ \varepsilon_o$ and $\varepsilon_{si} = 11.7\ \varepsilon_o$. Use the graph to calculate:



2.1 The oxide thickness t_{ox}.

	expression	value & units
$t_{ox} = \frac{\varepsilon_{ox}}{C_{ox}} = \frac{3.9\varepsilon_o}{C_{ox}} = \frac{11.7 \cdot 8.85\ 10^{-14} F/cm}{2.3 \cdot 10^{-15} F/\mu m^2}$		$0.015\mu m$

2.2 The maximum depth of the depletion region (*What is the V_{GB} voltage that yields the maximum depth of the depletion region?*).

$\frac{1}{C_{min}} = \frac{1}{C_{ox}} + \frac{1}{C_{depl}} \Rightarrow \frac{1}{C_{depl}} = \frac{1}{C_{min}} - \frac{1}{C_{ox}}$

$C_{depl} = \frac{\varepsilon_{si}}{X_{dmax}}$

	expression	value and units
$x_{dmax} = \varepsilon_{si}\left(\frac{1}{C_{min}} - \frac{1}{C_{ox}}\right) = 11.7\varepsilon_o\left(\frac{1}{0.3C_{ox}} - \frac{1}{C_{ox}}\right)$		$0.105\mu m$

V_{GB} when maximum depletion depth is achieved = 0.6V as shown in figure

2.3 The doping concentration of acceptors (N_A) in the channel.

$V_{FB} = -(\varphi_n^+ + \varphi_p) \Rightarrow \varphi_p = \overbrace{-V_{FB} - \varphi_n^+}^{-0.42}$

$\varphi_p = -60mV \cdot \log_{10}\frac{N_A}{n_i} \Rightarrow N_A = n_i \cdot 10^{-\frac{\varphi_p}{60mV}}$ (n_i: $10^{10}/cm^3$)

	expression	value and units
$N_A = n_i\ 10^{\frac{(V_{FB}+\varphi_n^+)}{60mV}}$		$10^{17}/cm^3$

2.4 The charge density of the inversion layer (in Cb/cm²) when V_{GB} is 2V.

Above V_{tn}, structure acts as a linear capacitor with value = C_{ox}.

$C_{ox} = -\frac{\Delta Q}{\Delta V} \Rightarrow \Delta Q = -C_{ox} \cdot \Delta V = -C_{ox}(2V - V_{tn})$

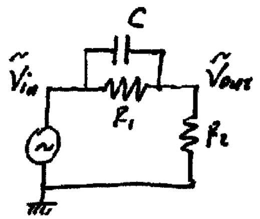
(ΔQ: inversion charge; ΔV: voltage above threshold)

	expression	value and units
$Q_i = -C_{ox}(V_{GB} - V_{tn})$ (C_{ox} = 2.3 fF/μm², $V_{GB} - V_{tn}$ = 1.4 Volts)		$-3.22\ fCb/\mu m^2$

(− since inversion layer is made of electrons)

Problem 3 of 4: Answer each question briefly and clearly. (20 points)

Consider the circuit below. Assume $R_1 = 1M\Omega$, $R_2 = 100\Omega$, and $C = 1pF$.



3.1. Write the transfer function v_{out}/v_{in}.

Classic voltage divider:

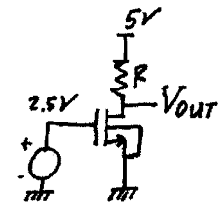
$$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{R_2}{R_2 + \left(\frac{1}{R_1} + j\omega C\right)^{-1}} = \frac{R_2}{R_1+R_2}\left[\frac{1+j\omega R_1 C}{1+j\omega (R_1 \| R_2) C}\right]$$

3.2 Calculate the gain at DC, the gain at extremely high frequencies (ω = infinity), and the numerical values of any poles and/or zeros that this function has.

expression	value
Gain at DC = $20\log\left(\frac{R_2}{R_1+R_2}\right)$	$\simeq -80dB$ -or- $\sim 1\times10^{-4}$
Gain at ω = infinity = $20\log(1)$ ← cap "shorts" R_1	0 dB
Pole 1 = $1/(R_1 \| R_2)C$ ~~Pole~~ 2 = function has only one pole... ~~...~~	10 Grad/sec
Zero 1 = $1/R_1 C$ ~~Zero~~ 2 = function has only one zero... ...	10^6 rad/sec

Problem 4 of 4: Answer each question briefly and clearly. (20 points)

You are given an nmos transistor, connected as shown.



4.1 Find an expression for W so that the output voltage V_{out} is 2.5V. Assume $\lambda_n = 0.1V^{-1}$ when L = 1µm, and it is otherwise proportional to 1/L.

transistor in saturation since $V_{DS} > V_{GS} - V_{Tn}$.

$$I_D = K_P \frac{W}{2L}(V_{GS}-V_{Tn})^2(1+\lambda_n' V_{DS})$$

$$\lambda_n' = \lambda_n \cdot \frac{\mu m}{L}$$

$$I_D = \frac{5V - V_{OUT}}{R}$$

expression

$$W = \frac{(5V - V_{OUT})/R}{\frac{K_P}{2L}(V_{GS}-V_{Tn})^2\left(1+\lambda_n \frac{\mu m}{L} V_{DS}\right)}$$

4.2 Assume $R = 10k\Omega$, $\mu_n C_{ox} = 50\mu A/V^2$, $V_{Tn} = 1V$, $\lambda_n = 0.1V^{-1}$ when L = 1µm, and it is otherwise proportional to 1/L. Find W/L when L is 2µm, and when L is 4µm.

$$\frac{W}{L} = \frac{(5V - V_{OUT})/R}{\frac{K_P}{2}(V_{GS}-V_{Tn})^2\left(1+\lambda_n \frac{\mu m}{L} V_{DS}\right)} =$$

$$= \frac{2\cdot 250\cdot 10^{-6} A}{50\mu A/V^2 (1.5V)^2\left(1+0.1 \frac{\mu m}{L}\cdot 2.5\right)} = \frac{10}{1.5^2\left(1+\frac{0.1}{L}\cdot 2.5\right)}$$

value

W/L when L = 2µm = 4.44/1.125 = 3.95

W/L when L = 4µm = 4.44/1.0625 = 4.18