

**University of California at Berkeley**  
**College of Engineering**  
**Dept. of Electrical Engineering and Computer Sciences**

**EE 105 Final Examination**

Spring 2002

Prof. Roger T. Howe

May 18, 2002

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Your Name (Last, First)

**Guidelines**

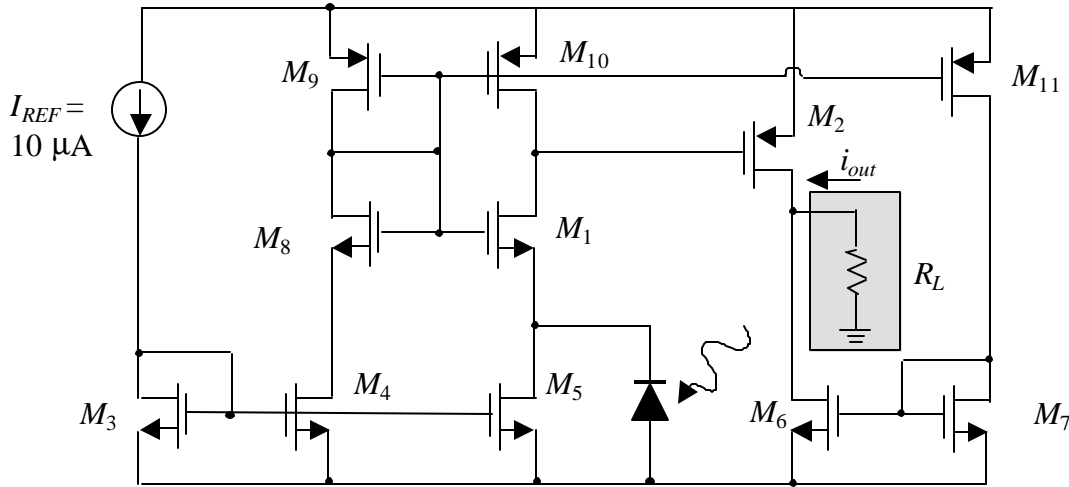
Closed book and notes; three 8.5" x 11" page (both sides) of *your own notes* is allowed.  
You may use a calculator.  
Do not unstaple the exam.  
Show all your work and reasoning on the exam in order to receive full or partial credit.  
You have 180 minutes (3 hrs.); use your time wisely. Good luck!

**Score**

Problem	Points Possible	Score
1	30	
2	25	
3	25	
4	20	
<b>Total</b>	100	

1. CMOS two-stage photocurrent amplifier [30 points]

$V^+ = 1.5 \text{ V}$



Given:

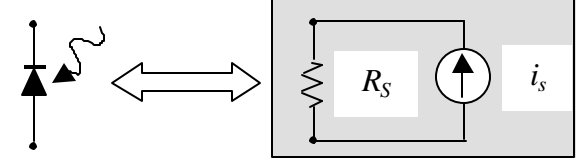
$V^- = -1.5 \text{ V}$

$\mu_n C_{ox} = 50 \mu\text{A}/\text{V}^2$ ,  $V_{Tn} = 1.0 \text{ V}$ ,  $\lambda_n = 0.05 \text{ V}^{-1}$

$\mu_p C_{ox} = 25 \mu\text{A}/\text{V}^2$ ,  $V_{Tp} = -1.0 \text{ V}$ ,  $\lambda_p = 0.05 \text{ V}^{-1}$

Neglect the backgate effect.

Illuminated Photodiode Model:



$M_1: (W/L)_1 = 256/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_2: (W/L)_2 = 256/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_3: (W/L)_3 = 16/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_4: (W/L)_4 = 32/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_5: (W/L)_5 = 256/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_6: (W/L)_6 = 128/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_7: (W/L)_7 = 32/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_8: (W/L)_8 = 32/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_9: (W/L)_9 = 64/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$M_{10}: (W/L)_{10} = 512/1 \text{ (}\mu\text{m}/\mu\text{m)}$

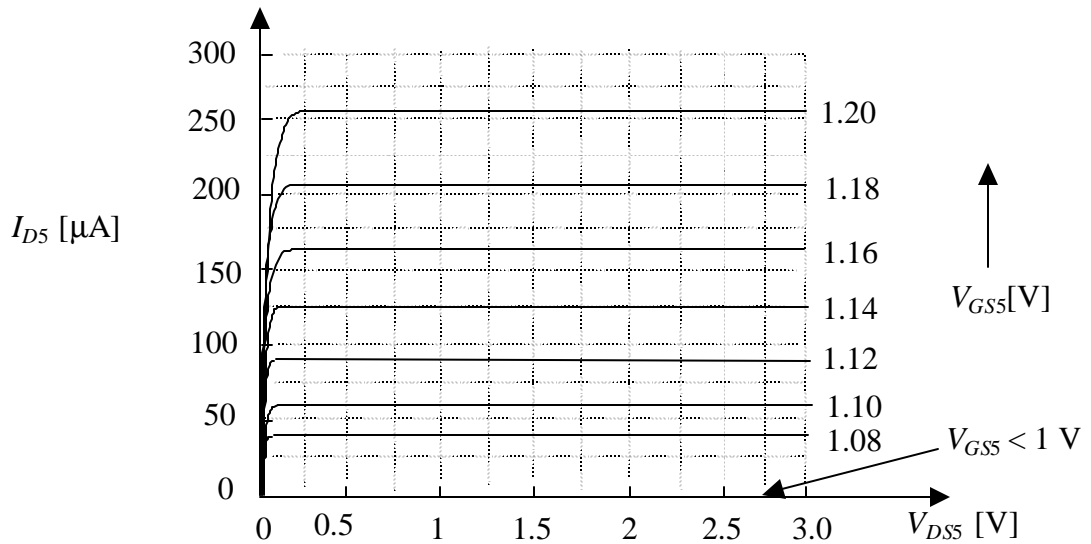
$M_{11}: (W/L)_{11} = 64/1 \text{ (}\mu\text{m}/\mu\text{m)}$

$R_S = 100 \text{ k}\Omega$   
 $R_L = 100 \text{ k}\Omega$

(a) [3 pts.] Find the numerical values of the DC drain currents for transistors  $M_1$ ,  $M_{10}$ , and  $M_2$ . You can assume that all transistors are saturated, that the DC output voltage is  $V_{OUT} = 0 \text{ V}$ , and that channel length modulation can be neglected.

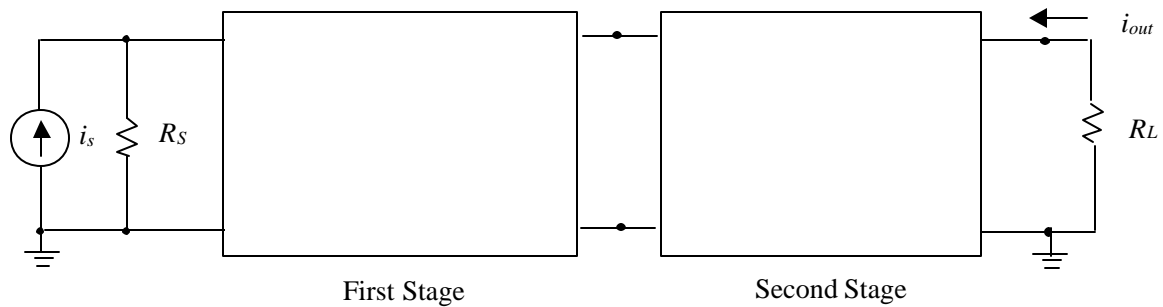
$I_{D1} = \underline{\hspace{2cm}} \mu\text{A}$	$-I_{D10} = \underline{\hspace{2cm}} \mu\text{A}$	$-I_{D2} = \underline{\hspace{2cm}} \mu\text{A}$
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- (b) [3 pts.] Locate the bias point  $Q$ , which is the point  $I_{D5}(V_{GS5}, V_{DS5})$ , for transistor  $M_5$  on the family of curves on the plot below. Select the nearest  $V_{GS5}$  curve from those given on the plot.



- (c) [5 pts.] Redraw the circuit with non-ideal (“gray circle”) current supplies and any bias current sources or voltage sources for each amplifier stage. Label with the numerical values of the DC currents  $I_{SUP}$  and  $I_{BIAS}$  and the DC voltage  $V_{DC}$ . There is no need to evaluate the resistances  $r_{oc}$ . *Hint*: your circuit should contain two current supplies, one bias current, and one bias voltage.

- (d) [4 pts.] Identify the two-port type for each amplifier stage (e.g., CS, CG, or CD) and fill in the appropriate circuit model (e.g., current buffer, transconductance amp, etc.). You should include the elements (e.g.,  $R_{in}$ ,  $R_{out}$ , gain element) for each stage in your circuit. There is no need to evaluate any of the parameters for this part.



- (e) [3 pts.] Find the numerical value of the input resistance  $R_{in}$  of this amplifier in Ohms. You should make reasonable approximations to simplify the calculations.

$R_{in} = \text{_____} \Omega$

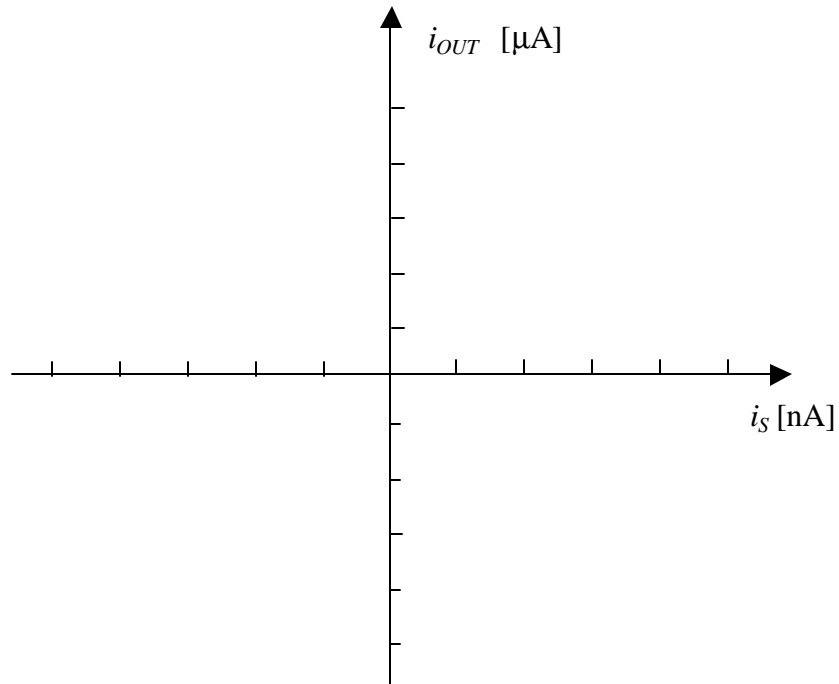
- (f) [5 pts.] Find the numerical value of  $A_i$  (the low-frequency short-circuit current gain -- a two-port parameter.) Again, you should make reasonable approximations in order to save time and pencil lead!

$$A_i = \underline{\hspace{2cm}}$$

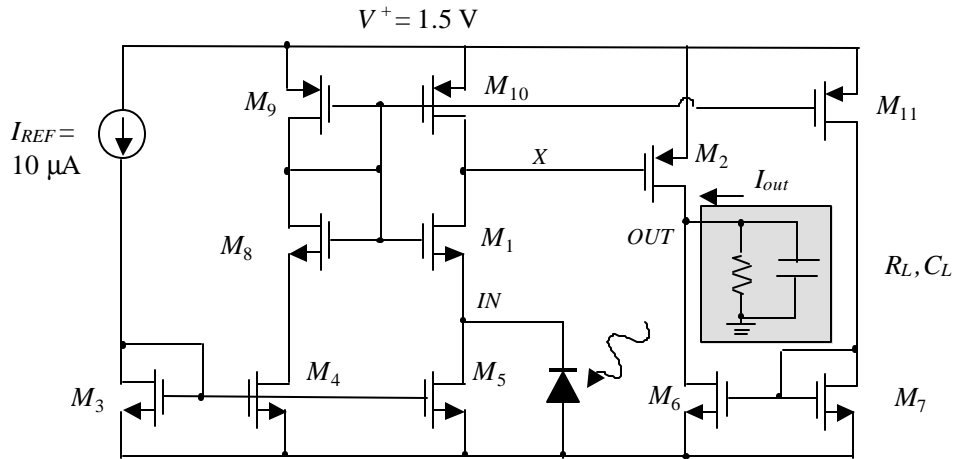
- (g) [4 pts.] Find the maximum *amplitude*  $\hat{i}_{OUT,MAX}$  of the output current  $i_{OUT}(t)$  in  $\mu\text{A}$  which avoids clipping. Note that the value of the load resistor is  $R_L = 100 \text{ k}\Omega$ .

$$\hat{i}_{OUT,MAX} = \underline{\hspace{2cm}} \mu\text{A}$$

- (h) [3 pts.] Sketch the transfer curve  $i_{OUT}$  vs.  $i_S$  where the output current is in  $\mu\text{A}$  and the input current is in  $\text{nA} = 10^{-9} \text{ A}$ . Consider the input source to be “large signal” for this part and neglect  $R_S$ . Provide your own appropriate scale -- your sketch should be consistent with your answers to parts (f) and (g). If you couldn't solve these parts, you can assume that the overall short-circuit current gain is 75 and that  $-1.2 \text{ V} < v_{OUT} < 0.9 \text{ V}$  (not the correct answers, of course).



2. Frequency Response of a CMOS Photocurrent Amplifier [25 points]



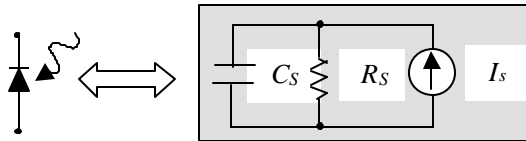
Given:

$$V^- = -1.5 \text{ V}$$

Illuminated Photodiode Model:

$$\mu_n C_{ox} = 50 \text{ } \mu\text{A}/\text{V}^2, V_{Tn} = 1.0 \text{ V}, \lambda_n = 0.05 \text{ V}^{-1}$$

$$\mu_p C_{ox} = 25 \text{ } \mu\text{A}/\text{V}^2, V_{Tp} = -1.0 \text{ V}, \lambda_p = 0.05 \text{ V}^{-1}$$



$$M_1: (W/L)_1 = 256/1 \text{ (}\mu\text{m}/\mu\text{m)}$$

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$$M_8: (W/L)_8 = 32/1 \text{ (}\mu\text{m}/\mu\text{m)}$$

$$M_9: (W/L)_9 = 64/1 \text{ (}\mu\text{m}/\mu\text{m)}$$

$$M_{10}: (W/L)_{10} = 512/1 \text{ (}\mu\text{m}/\mu\text{m)}$$

$$R_S = 100 \text{ k}\Omega$$

$$C_S = 100 \text{ fF}$$

$$R_L = 100 \text{ k}\Omega$$

$$C_L = 100 \text{ fF}$$

Device Capacitance Data:  $C_{ox} = 2 \text{ fF}/\mu\text{m}^2$ ,  $C_{ov} = 0.1 \text{ fF}/\mu\text{m}$ ,  $L_{diff} = 2 \text{ } \mu\text{m}$ ,  
 NMOS:  $C_{db} = 0.5 \text{ fF}/\mu\text{m}^2$ , PMOS:  $C_{db} = 0.3 \text{ fF}/\mu\text{m}^2$

- (a) [4 pts.] What is the numerical value of the total capacitance  $C_{in}$  between the input node  $IN$  and small-signal ground (in fF)? You can consider any node connected to a DC voltage source through a diode-connected MOSFET to be effectively at small-signal ground. Also, assume  $V_{SB1} = 0 \text{ V}$ . List all relevant capacitances symbolically (e.g.,  $C_{gs1}$ ) before calculating their values.

$$C_{in} = \underline{\hspace{2cm}} \text{ fF}$$

- (b) [3 pts.] What is the open-circuit time-constant  $\tau_{in}$  associated with  $C_{in}$  (in nanoseconds,  $10^{-9}$  s)? If you couldn't solve problem 1(e), you can assume that the input resistance  $R_{in} = 2.5 \text{ k}\Omega$  for this part.

$\tau_{in} = \underline{\hspace{2cm}} \text{ ns}$
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- (c) [5 pts.] By using Miller's theorem and including all relevant capacitors, what is the total capacitance  $C_X$  between node  $X$  and small-signal ground (in fF)? You can consider any node connected to a DC voltage source through a diode-connected MOSFET to be effectively at small-signal ground. List all relevant capacitances symbolically (e.g.,  $C_{gd1}$ ) before calculating their values.

$C_X = \underline{\hspace{2cm}} \text{ fF}$
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- (d) [2 pts.] What is the open-circuit time-constant  $\tau_X$  associated with  $C_X$  (in nanoseconds,  $10^{-9}$  s)? If you couldn't solve problem 1(f), you can assume that the short-circuit current gain  $A_i = 105$ .

$$\tau_X = \text{_____ ns}$$

- (e) [3 pts.] What is the numerical value of the total capacitance  $C_{out}$  between the output node *OUT* and small-signal ground (in fF)? You can consider any node connected to a DC voltage source through a diode-connected MOSFET to be effectively at small-signal ground. List all relevant capacitances symbolically (e.g.,  $C_{gd6}$ ) before calculating their values.

$$C_{out} = \text{_____ fF}$$

- (f) [3 pts.] What is the numerical value of the open-circuit time constant  $\tau_{out}$  for the capacitance between the output node *OUT* and small-signal ground (in nanoseconds,  $10^{-9}$  s)?

$$\tau_{out} = \text{_____ ns}$$

- (g) [3 pts.] What is the  $-3\text{dB}$  frequency  $\omega_1$  for this amplifier in Mrad/s, according to the open-circuit time constant method? If you couldn't do parts (a)-(f), you can assume here that  $C_{in} = 120 \text{ fF}$ ,  $C_X = 225 \text{ fF}$ , and  $\tau_{out} = 35 \text{ ns}$  – all incorrect answers, of course.

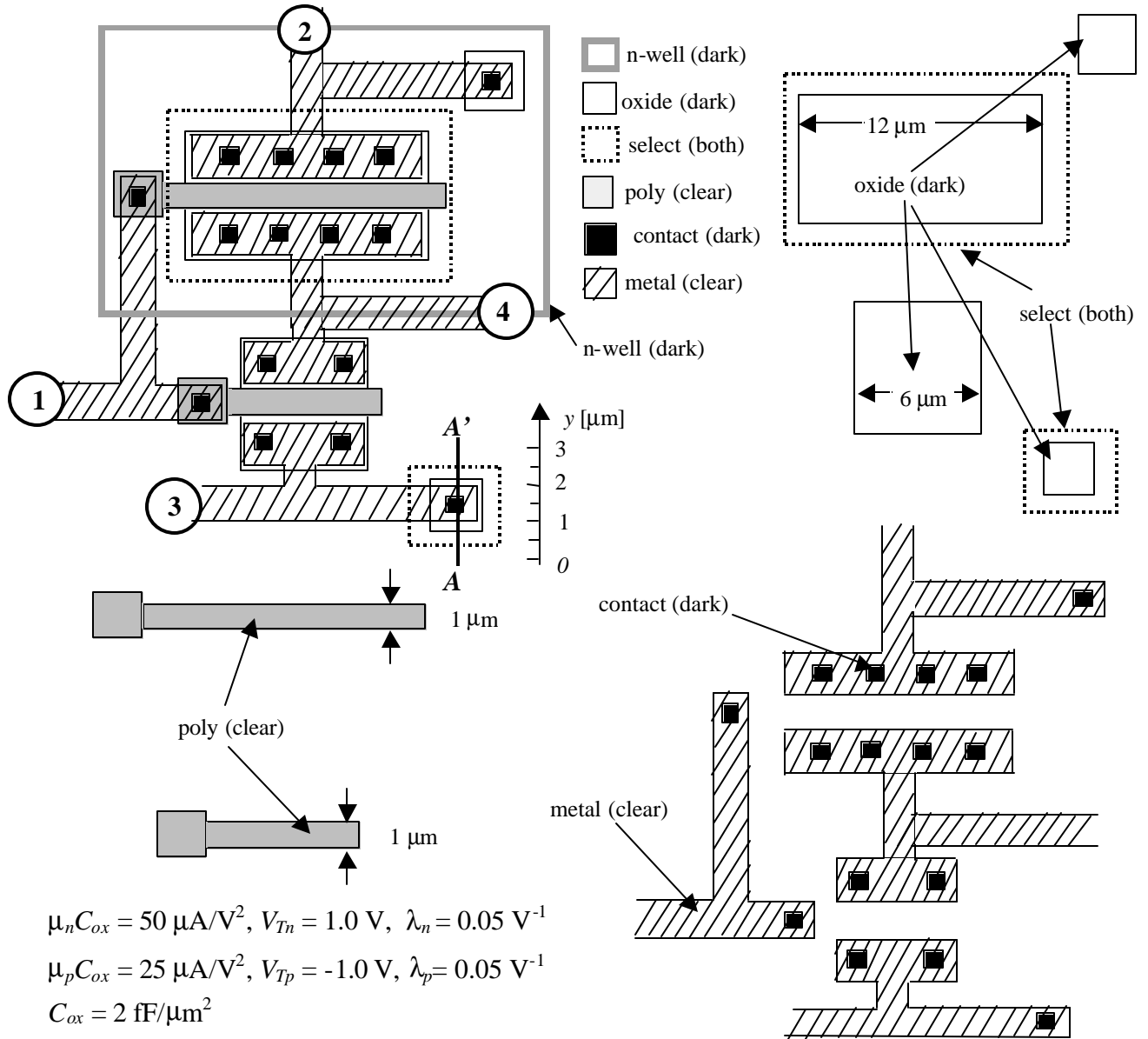
$\omega_1 = \underline{\hspace{2cm}} \text{Mrad/s}$

- (h) [2 pts.] The bandwidth is not high enough, so you decide to add a 3<sup>rd</sup> stage. Which CMOS 3<sup>rd</sup> stage, if any, would improve the bandwidth of this amplifier? Circle one answer and justify it.

*Circle one:*    1. CS                    2. CG                    3. CD                    4. None

Justify your choice in one or two sentences:

3. New CMOS amplifier configuration [25 points]



$$\mu_n C_{ox} = 50 \mu\text{A}/\text{V}^2, V_{Tn} = 1.0 \text{ V}, \lambda_n = 0.05 \text{ V}^{-1}$$

$$\mu_p C_{ox} = 25 \mu\text{A}/\text{V}^2, V_{Tp} = -1.0 \text{ V}, \lambda_p = 0.05 \text{ V}^{-1}$$

$$C_{ox} = 2 \text{ fF}/\mu\text{m}^2$$

Node Voltages:

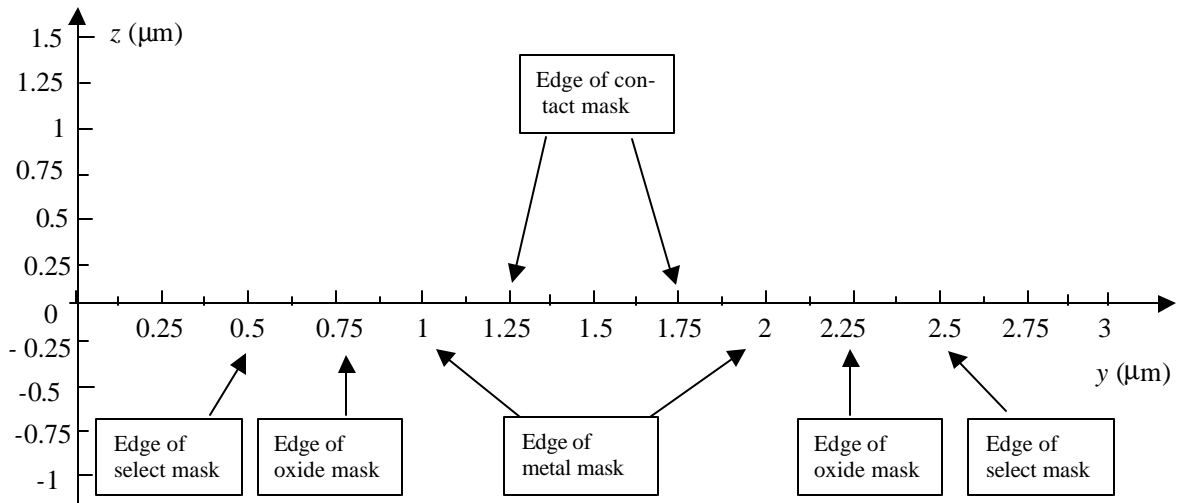
1: Input, 2:  $V_{DD} = 2.5 \text{ V}$ , 3:  $V_{SS} = 0 \text{ V}$ ; 4: Output

**n-well CMOS process:** starting material: boron-doped silicon, conc.  $1 \times 10^{16} \text{ cm}^{-3}$

1. Deposit 500 nm of thermal silicon dioxide and pattern using the **well mask**
2. High-energy phosphorus implant; rapid-thermal anneal to form 2  $\mu\text{m}$ -deep well with only a 0.5  $\mu\text{m}$ -wide lateral spread of phosphorus under the well mask pattern (through the end of the process)
3. Etch off oxide, deposit 500 nm of silicon dioxide, pattern with **oxide mask**
4. Grow a 2.5 nm-thick gate oxide

5. Deposit 250 nm of heavily phosphorus-doped polysilicon and pattern with **poly mask**
6. Implant boron using photoresist patterned with the **select mask (dark field)** as an implant mask; strip resist.
7. Implant arsenic using photoresist patterned with the **select mask (clear field)** as an implant mask; strip resist.
8. Rapid-thermal anneal to form 0.25  $\mu\text{m}$ -deep regions that spread laterally by  $L_D = 0.1 \mu\text{m}$ . Net doping conc. for both n and p type source/drain regions:  $5 \times 10^{18} \text{ cm}^{-3}$ . The polysilicon layer is n-type at the end of this step.
9. Deposit 500 nm of silicon dioxide and etch 502.5 nm of oxide with the **contact mask**
10. Deposit 500 nm of aluminum and pattern using the **metal mask** (clear field).

- (a) [5 pts.] Draw the cross section along **A-A'** on the graph below. The locations of the mask edges have been indicated on the  $x$  axis scale for your convenience. Label any ion-implanted regions in the substrate.



- (b) [4 pts.] Draw the schematic of the circuit corresponding to this layout, including the connections for all four terminals and the  $(W/L)$  ratios of any MOSFETs in the circuit. Note that nodes 1-4 are defined below the schematic on p. 11.

- (c) [4 pts.] Find the numerical value of the DC bias voltage  $V_{BIAS}$  between nodes 1 and 3 that is needed for the DC output voltage (node 4) to equal  $V_{DD}/2 = 2.5/2 \text{ V} = 1.25 \text{ V}$ . MOSFET parameters are listed below the schematic on p. 11. You can neglect channel-length modulation for this part.

$$V_{BIAS} = \text{_____ V}$$

- (d) [4 pts.] Find the numerical value of the short-circuit transconductance  $G_m$  for this amplifier, when biased at the operating point specified in part (c). If you couldn't solve part (c), you can assume that any MOSFETs in the circuit are biased at a drain current  $|I_D| = 20 \mu\text{A}$ .

$$G_m = \text{_____ } \mu\text{S}$$

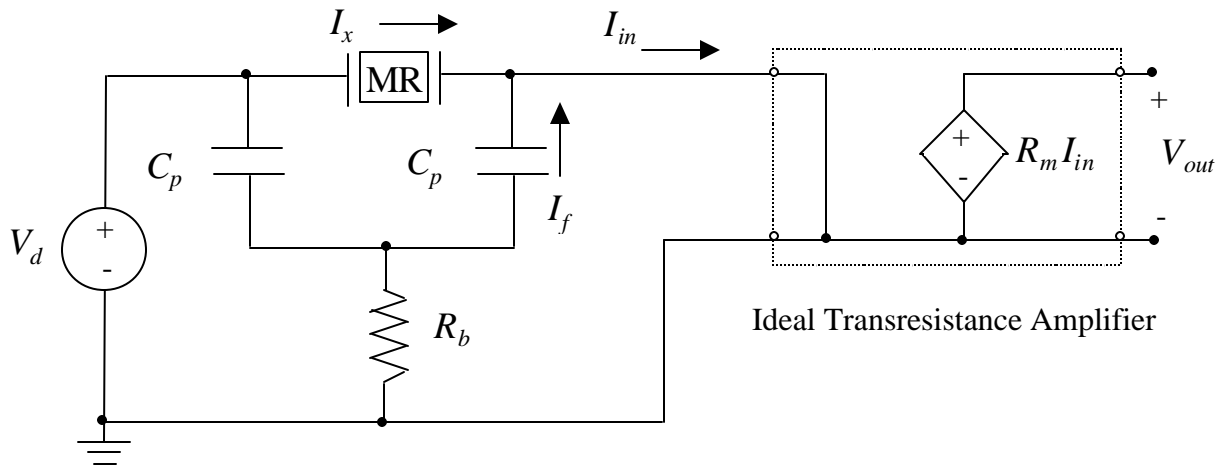
- (e) [4 pts.] Find the numerical value of the output resistance of this amplifier, when biased at the operating point specified in part (c). If you couldn't solve part (c), you can assume that any MOSFETs in the circuit are biased at a drain current  $|I_D| = 20 \mu\text{A}$ .

$$R_{out} = \text{_____k}\Omega$$

- (f) [4 pts.] What is the input capacitance  $C_{in}$  of this amplifier in fF when biased at the operating point specified in part (c), for the case where  $R_L$  is infinity? The input capacitance is the total capacitance (including any Miller capacitance) between the input (node 1) and small-signal ground. ) If you couldn't solve part (c), you can assume that any MOSFETs in the circuit are biased at a drain current  $|I_D| = 20 \mu\text{A}$ .

$$C_{in} = \text{_____fF}$$

4. Feedthrough model [20 points]



This circuit model represents substrate feedthrough around a MEMS resonator (MR).

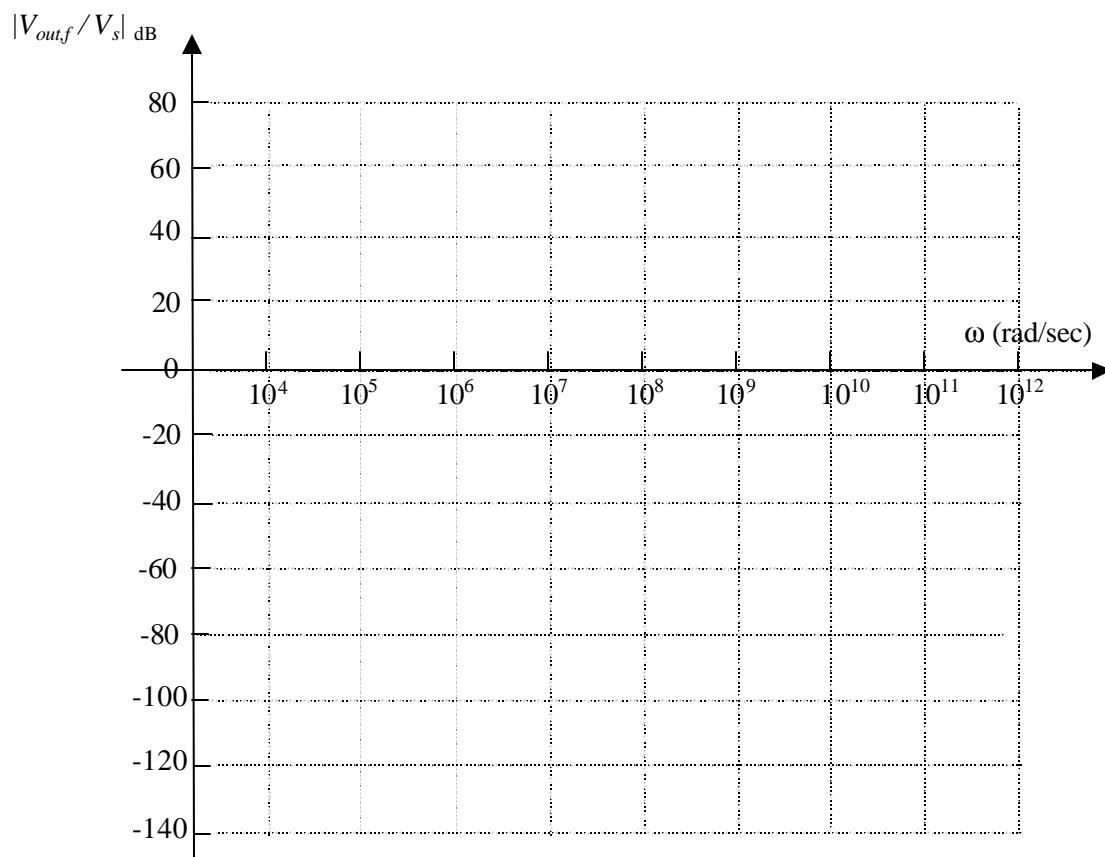
- (a) [5 pts.] Find an expression for the phasor feedthrough current  $I_f$  in terms of the drive voltage  $V_d$ . Do *not* simplify the expression for this part.
- (b) [5 pts.] Find an expression for the output voltage  $V_{out,f}$  due to  $I_f$  (that is, with  $I_x$  set to 0 A). Your answer should be expressed as  $V_{out,f} / V_d$  and must be reduced to the form of one pole and a numerator proportional to  $\omega^2$ .

- (c) [5 pts.] On the graph below, plot the magnitude Bode Plot for the transfer function  $V_{out,f} / V_d$ . The parameters are:  $C_p = 500$  fF,  $R_b = 500 \Omega$ , and  $R_m = -1$  M $\Omega$ .

If you couldn't solve part (b), you can plot the transfer function:

$$\frac{V_{out,f}}{V_s} = \frac{K \omega^2}{1 + j\omega / \omega_1}$$

where  $K = 5 \times 10^{-16} \text{ s}^2$  and  $\omega_1 = 8 \times 10^8 \text{ rad/s}$ .





- (d) [5 pts.] If a drive voltage waveform is  $v_d(t) = 100 \text{ mV} \cos[(10 \text{ Mrad/s})t]$  is applied to the input, what is the output voltage waveform due to feedthrough,  $v_{out,f}(t)$ ? Use your results from parts (b) and (c), or the alternative transfer function given in part (c)