



First Midterm Exam, February 23, 2001 (53 Minutes)

Print Your Name: _____ **Solutions (see the following pages)**

Sign Your Name: _____

Use the following parameters for MOS devices.

$$I_{Dn} = \left(\frac{W}{L}\right)_p m_n C_{oxp} [V_{GS} - V_{Tn} - (V_{DS}/2)] (1 + I_n V_{DS}) V_{DS}$$

$$I_{Dn} = \frac{1}{2} \left(\frac{W}{L}\right)_n m_n C_{oxn} (V_{GS} - V_{Tn})^2 (1 + I_n V_{DS})$$

$$-I_{Dp} = \left(\frac{W}{L}\right)_p m_p C_{oxp} [V_{SG} + V_{Tp} - (V_{SD}/2)] (1 + I_p V_{SD}) V_{SD}$$

$$-I_{Dp} = \frac{1}{2} \left(\frac{W}{L}\right)_p m_p C_{oxp} (V_{SG} + V_{Tp})^2 (1 + I_p V_{SD})$$

NMOS	PMOS
$\mu_n C_{ox} = 50 \mu A/V^2$	$\mu_p C_{ox} = 25 \mu A/V^2$
$V_{T0n} = 1.0V$	$V_{T0p} = -1.0V$
$\gamma_n = 0.6V^{1/2}$	$\gamma_p = 0.6V^{1/2}$
$\epsilon_n = (0.1/L)V^{-1} L$ in μm	$\epsilon_p = (0.1/L)V^{-1} L$ in μm
$\phi_p = -0.42$	$\phi_n = 0.42$

Assume $L = 2 \mu m$

Here are a few fundamental constants and room temperature values.

$$e_o = 8.85 \times 10^{-14} F/cm \quad e_{r,ox} = 3.9 \quad e_{r,si} = 11.7$$

$$V_{th} = 0.026eV \quad n_i^2 = 2 \times 10^{20} \quad K = 1.38 \times 10^{-23} J/K$$

$$q = 1.6 \times 10^{-19} C$$

mobility	10^{15}	10^{16}	10^{17}	units
μ_n	1400	1200	750	$cm^2/V\cdot s$
μ_p	500	400	350	$cm^2/V\cdot s$

Problem	Possible	Score
I	20	
II	30	
III	20	
IV	30	
Total	100	

I. (20 Points) Threshold Voltage

An NMOS process produces a device with a threshold voltage $V_{Tn} = 1V$ and $\phi_p = -0.42$. Complete the table below that describes the percentage change in the electrical properties listed that is produced by the change shown in the columns for the oxide thickness and doping. You must briefly explain your listed percentages with footnotes 1-6 as indicated.

Electrical Parameter	Oxide thickness t_{OX}		Substrate doping N_A	
	doubles		doubles	
$2\phi_p$	1	0%	4	+4.3%
C_{OX}	2	-50%	5	0%
$Q_{B,MAX}/C_{OX}$	3	+100%	6	+44%

1) No effect

2) C_{OX} is inverse with t_{OX}

3) C_{OX} decreases causes inverse Q/C_{OX} increase

4) (2) $26mV \ln(2) = 36mV$ or 4.3% of 840 mv

5) No effect

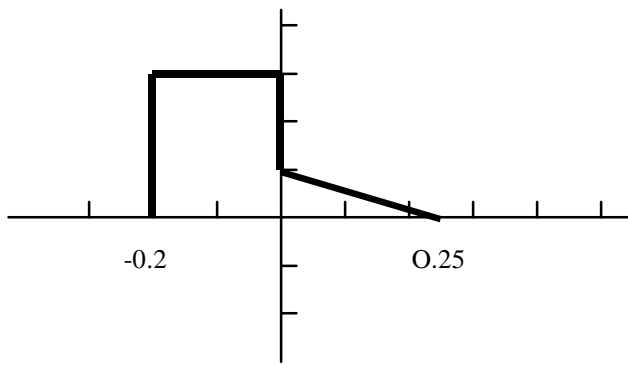
6) $\text{Sqrt}(2 * (1.043)) = 1.44$ or 44%

II. (30 Points) Electrostatic and Mobile Carrier Analysis

An NMOS device is made in a p-type substrate doped $N_a = 10^{16}$ and an oxide thickness of $0.2 \mu\text{m}$. A region $0.25 \mu\text{m}$ deep under the gate is depleted.

- a) (15 Points) Sketch the electric field versus position from the gate electrode. Give the quantitative value of the maximum electric field.

$$E_{MAX} = \frac{qN_a x_d}{\epsilon_{OX}} = \frac{1.6 \times 10^{-19} \cdot 1 \times 10^{16} \cdot 0.25 \times 10^{-4}}{3.9 \cdot 8.85 \times 10^{-14}} = 1.16 \times 10^5 \text{ V/cm}$$



- b) (15 Points) Find the local potential and the density of mobile electrons at the silicon-oxide interface.

$$f(x) = A(x - x_p)^2$$

$$E(x) = -\frac{\partial f}{\partial x} = 2A(x - x_p)$$

$$E_{x=0} = 2Ax_p$$

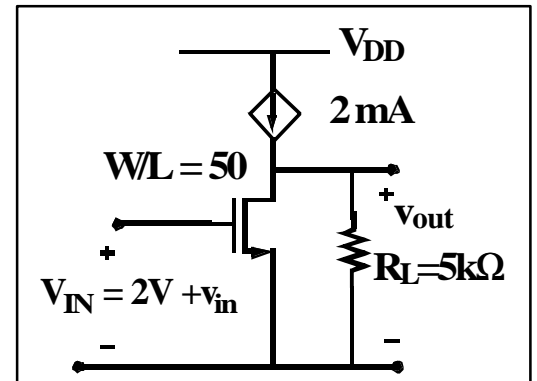
$$f(x) = \frac{E_{x=0}}{2x_p} (x - x_p)^2$$

$$f(0) = \frac{E_{x=0} x_p}{2} = \frac{1.16 \times 10^5 \cdot 10^{16}}{(11.7/3.9) \cdot 2} = 483 \text{ mV}$$

$$n(0) = \frac{2 \times 10^{20}}{10^{16}} \exp\left(\frac{483}{26}\right) = 2 \times 10^4 \cdot 1.16 \times 10^8 = 2.3 \times 10^{12}$$

III. (20 Points) MOS Circuits

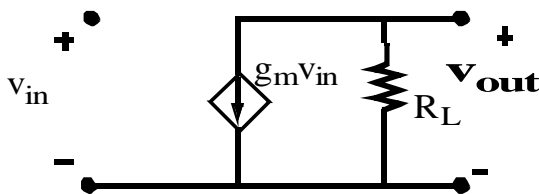
- a) (10 Points) Assuming the device is in saturation and $\lambda_n = 0$, find the large-signal quantity V_{OUT} .



$$I_D = 0.5 \cdot 50 \cdot 50 \cdot (2 - 1)^2 = 1.25 \text{ mA}$$

$$V_{OUT} = (2 \text{ mA} - 1.25 \text{ mA}) \cdot 5 \text{ k}\Omega = 3.75 \text{ V}$$

- b) (10 Points) Find the small-signal gain v_{OUT}/v_{IN} for the circuit in part a) when $\lambda_n = 0$.



$$g_m = 50 \cdot 50 \cdot (2 - 1) = 2.5 \text{ mS}$$

$$V_{OUT} = -g_m R_L = -2.5 \text{ mS} \cdot 5 \text{ k}\Omega = -12.5$$

IV. (30 Points) Advanced MOS Circuits

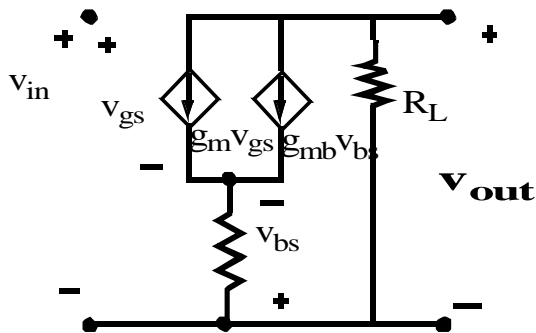
a) (7 Points) A resistor R_S is inserted as shown and has a value such that $V_S = 0.5V$. Find the large-signal quantity V_{OUT} when $\lambda_n = 0$.

$$V_T = 1 + 0.6(\sqrt{0.5 + 0.84} - \sqrt{0.84}) = 1.145V$$

$$I_D = 0.5 \cdot 50 \cdot 50 \cdot (2 - 0.5 - 1.145)^2 = 0.158mA$$

$$V_{OUT} = (2mA - 0.158mA) \cdot 5k\Omega = 9.21V$$

b) (14 Points) Draw the small-signal equivalent circuit of the circuit in part a) and give quantitative values for each element in the circuit when $\lambda_n = 0$.



$$g_m = 50 \cdot 50(2 - 0.5 - 1.145) = 0.89mS$$

$$g_{mb} = \frac{0.6 \cdot 0.89mS}{2 \cdot \sqrt{0.84 + 0.5}} = 0.23mS$$

$$R_S = \frac{0.5V}{0.158mA} = 3.16K\Omega$$

c) (9 Points) Write a sufficient set of equations to determine the small-signal gain v_{OUT}/v_{IN} when $\lambda_n = 0$. Solve for the gain if you have completed all other questions on this exam.

$$V_{in} = V_{gs} - V_{bs}$$

$$g_m v_{gs} + g_{mb} v_{bs} + \frac{v_{bs}}{R_S} = 0$$

$$V_{OUT} = -(g_m v_{gs} + g_{mb} v_{bs}) R_L$$

$$\frac{V_{OUT}}{V_{in}} = \frac{-(R_L / R_S)}{\left(1 + \frac{g_{mb}}{g_m} + \frac{1}{g_m R_S}\right)}$$

Use the second equation to find v_{gs} in terms of v_{bs} and then substitute in the first equation to get v_{bs} in terms of v_{in} . Now go backwards and get v_{gs} and then substitute both into the third equation.

