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UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits Fall 2009

MIDTERM EXAMINATION #3

11/24/2009
Time allotted: 75 minutes

NAME: ____________________ Solution

STUDENT ID#: ____________________

INSTRUCTIONS:

1. SHOW YOUR WORK. (Make your methods clear to the grader!)
2. Clearly mark (underline or box) your answers.
3. Specify the units on answers whenever appropriate.

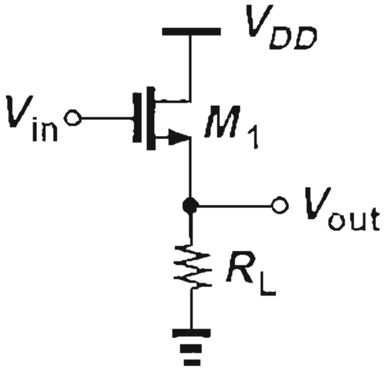
SCORE: 1 ________ / 16

2 ________ / 18

3 ________ / 16

Total ________ / 50

(d) [5 pt] What is the definition of cut-off frequency? Find out the expression for cut-off frequency for the following circuit in terms of MOSFET capacitances and gm.



Cutoff frequency is defined as the frequency at which

$$\left|\frac{I_{out}}{I_{in}}\right| = 1 \Rightarrow |g_m Z_{in}| = 1$$

$$Z_{in} = \frac{1}{\omega\left\{C_{gd} + C_{gs}\left(1 - {}^{V_{in}}/_{V_{out}}\right)\right\}} \; ; \quad \frac{V_{out}}{V_{in}} = \frac{\cancel{1/g_m}\, R_L}{1/g_m + R_L}$$

$$\cancel{\frac{1}{1+g_m R_L}}$$

$$\therefore 1 - \frac{V_{out}}{V_{in}} = 1 - \frac{R_L}{1/g_m + R_L}$$

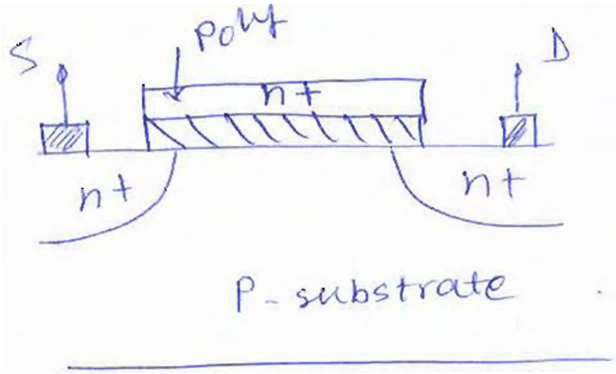
$$= \frac{1/g_m}{1/g_m + R_L} = \frac{1}{1 + g_m R_L}$$

$$g_m Z_{in} = \frac{g_m}{\omega\left\{C_{gd} + \frac{C_{gs}}{1+g_m R_L}\right\}} = 1$$

$$\therefore \boxed{f_T = \frac{g_m}{2\pi} \cdot \frac{1}{C_{gd} + C_{gs}/g_m R_L}}$$

1. MOSFET: Principle of Operation [16 pts]

   (a) [2 pts]Draw a schematic of a MOSFET with p-type substrate clearly showing the type of doping in source, drain and gate poly region.

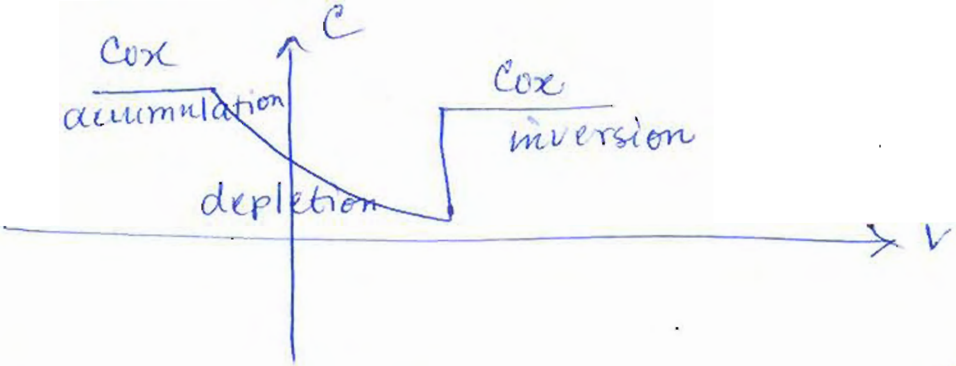


   (b) [2 pts] Draw the equivalent capacitive network clearly mentioning the physical origin of each of the capacitances.

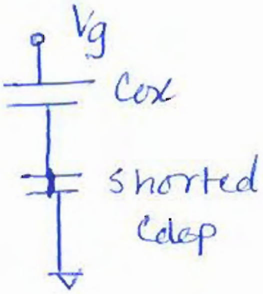
Cox = capacitance due to insulator /SiO_2

Cdep = depletion capacitance in the Bulk.

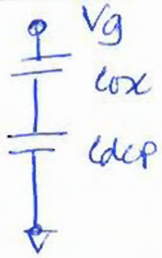
(c) [4 pts] Draw the C-V characteristic of a MOSFET with p-type substrate. Explain the different regions using the capacitance network from part (b).



acc

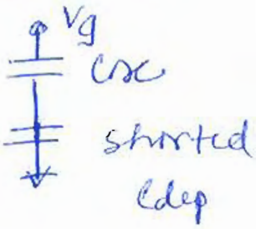


dep



$$C = \frac{C_{ox} C_{dep}}{C_{ox} + C_{dep}}$$

inversion



(d) [8 pts]

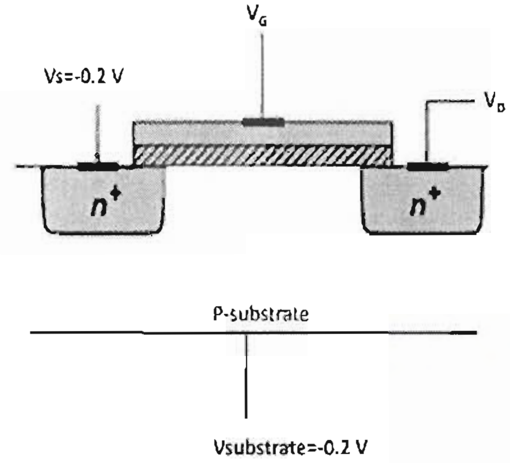
(i) [2 pts] Write down the equation of drain current as a function of gate and drain voltages in the linear and saturation region.

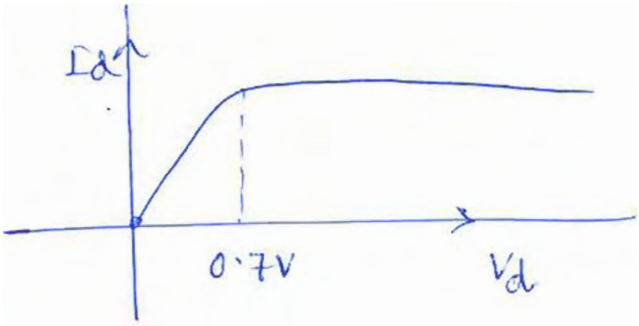
$$I_D = \begin{cases} \mu_n C_{ox} \frac{W}{L}\left[(V_{GS}-V_{tn})V_{DS} - \frac{V_{DS}^2}{2}\right]; & \text{linear} \\ \cancel{\tfrac{1}{2}}\ \mu_n C_{ox} \frac{W}{L} \frac{(V_{GS}-V_{tn})^2}{2}\left[1+\lambda\left(V_{DS}-V_{D,sat}\right)\right]; & \text{saturation} \end{cases}$$

(ii)[4 pts] Draw Id vs. Vd for the long channel MOSFET shown in the figure for the specified biasing conditions. Note that all the voltages are measured with respect to ground which is at zero voltage. In your plot, you must specify numerical values of all the relevant voltages. The current values need not be calculated numerically. Assume Vg=1 V.

Assume

$$V_{FB} + 2\phi_B + \frac{\sqrt{2q\varepsilon_{Si}N_A(2\phi_B)}}{C_{ox}} = 0.3\text{ V}$$





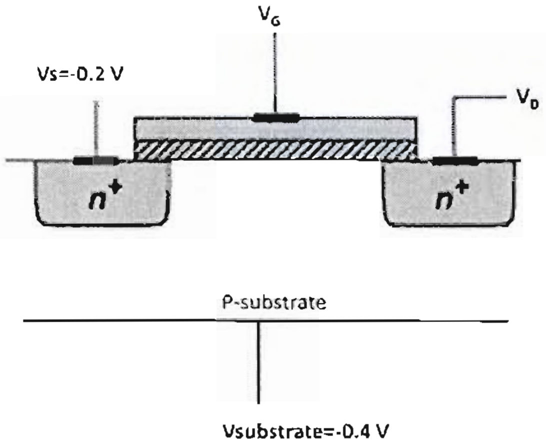
sat:

$V_{ds} > V_{gs} - V_{th}$

$V_d > V_g - V_{th}$

$V_g = 1\text{ V};\ V_{th} = 0{\cdot}3\text{ V}$

(iii)[2 pt] Show qualitatively how the Id-Vd will change if the substrate voltage is modified in the following way? Assume Vg=1 V.

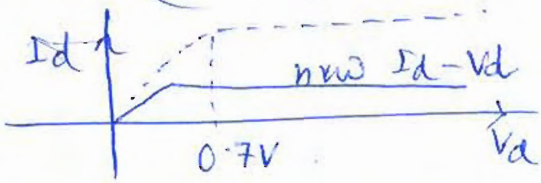


source junction is now reverse biased. Hence V_{th} goes up.

$\therefore V_{d_{sat}}^{new} = V_g - V_{th}^{new}$

Hence $V_{d_{sat}}^{new} < V_{d_{sat}}^{old}$

also $(V_{gs} - V_{th})^2$ is smaller



2. [18 pt] Operation of MOSFET and basic amplifiers.

(a) [2 pt} Considering that $I_{dsat} \propto \frac{1}{L-\Delta L}$, explain why channel length modulation does not affect the long channel MOSFETs.

$$I_{d,sat} \propto \frac{1}{L-\Delta L} \propto \frac{1}{L}\left(1+\frac{\Delta L}{L}\right)$$

$$\frac{\Delta L}{L} \ll 1$$

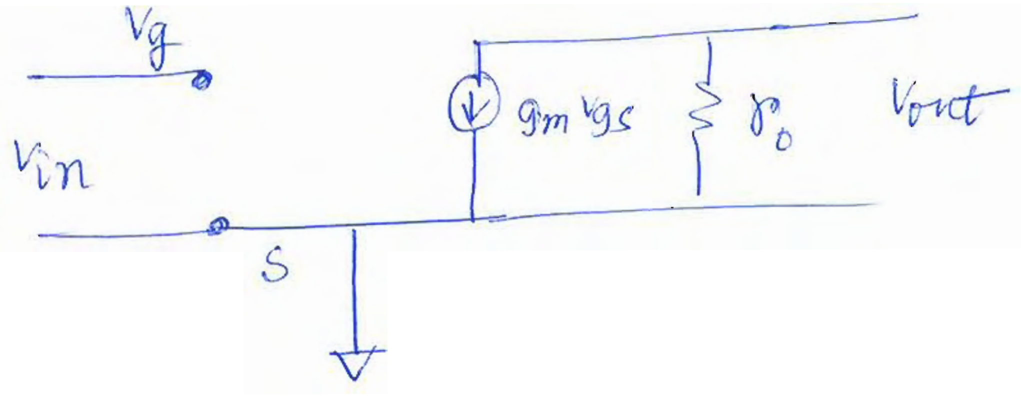
(b) [2 pt] The equation for current including channel length modulation is usually expressed as:

$$I_{D,sat} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\left[1+\lambda(V_{DS}-V_{D,sat})\right]$$

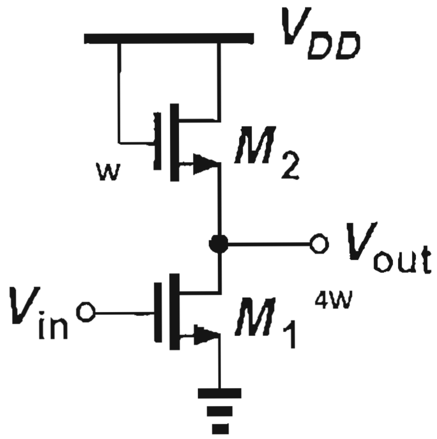
What relationship between Δ and (V_{DS}-$V_{D,sat}$) is assumed to derive this equation?

$$\Delta L \propto (V_{DS} - V_{D,sat})$$

(c) [2 pt] Draw the small signal equivalent of a CS amplifier with $\lambda \neq 0$.



(d) [4 pt] Consider the following circuit where two short channel MOSFETs of equal length have been arranged to give an amplifier. Approximate the total gain of this amplifier. Provide a numerical value.



$$A_v = -g_{m_1}\left(r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m_2}}\right)$$

$$\cong -g_{m_1} \times \frac{1}{g_{m_2}}$$

$$\cong -\sqrt{W_1/L_1} \times \sqrt{L_2/W_2}$$

$$= -\sqrt{\frac{W_1}{W_2}}$$

$$= -\sqrt{4}$$

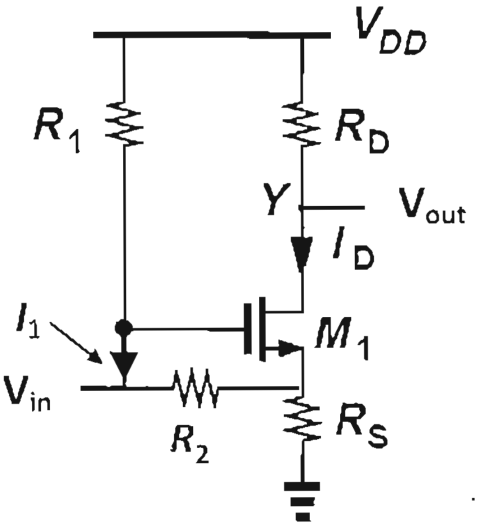
$$\boxed{A_v = -2}$$

(e) [6 pt] For the circuit shown below find out ($\lambda = 0.$)

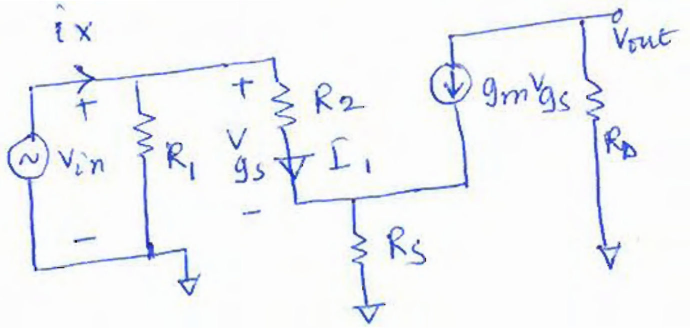
(i) $\beta = \frac{\partial I_D}{\partial I_1}$.

(ii) Small signal input impedance.

(iii) Small signal voltage gain.



$\Rightarrow$



(i) $\delta I_D = g_m v_{gs}$; $\delta I_1 = v_{gs}/R_2$

∴ $\boxed{\beta = \frac{\partial I_D}{\partial I_1} = g_m R_2}$

(ii) $V_{in} = V_{gs} + R_S\left[g_m V_{gs} + \frac{V_{gs}}{R_2}\right]$

$= V_{gs}\left[1 + \frac{R_S}{R_2} + g_m R_S\right]$ — Ⓐ

$i_x = \frac{V_{in}}{R_1} + \frac{V_{gs}}{R_2}$

$= V_{in}/R_1 + V_{in}/R_2[1 + R_S/R_2 + g_m R_S]$

$= V_{in}/R_1 + \frac{V_{in} \cdot R_2}{R_2 + R_S + \beta R_S} \cdot \frac{1}{R_2}$

∴ $R_{in} = \frac{V_{in}}{i_x} = \frac{1}{1/R_1 + \frac{R_2}{R_2 + (\beta+1) R_S} \cdot \frac{1}{R_2}}$

∴ $R_{in} = \frac{1}{1/R_1 + \frac{1}{R_2 + (\beta+1) R_S}}$

$\boxed{R_{in} = R_1 \parallel \{R_2 + (\beta+1) R_S\}}$

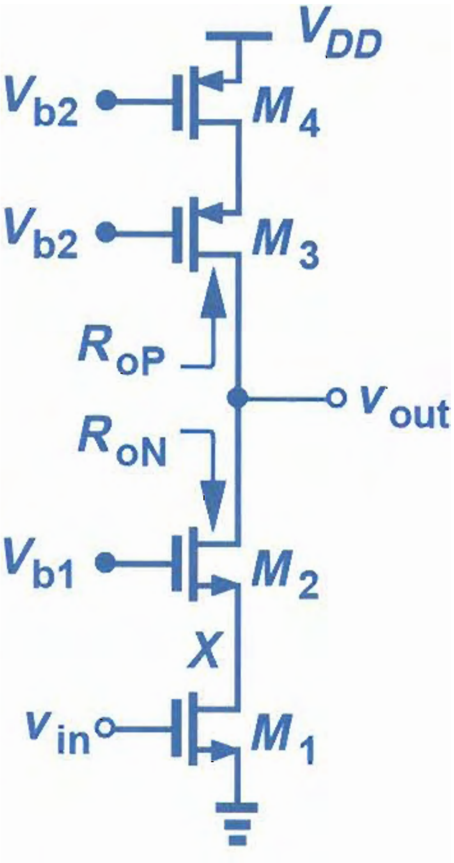
(iii) $A_v = \frac{v_{out}}{v_{in}}$

$= \frac{-g_m v_{gs} R_D}{V_{in}}$

$\boxed{A_v = \frac{-g_m R_D}{1 + \frac{R_S}{R_2} + g_m R_S}}$

3. [16 pt] MOSFET Cascodes, Current mirrors and Frequency Response.

(a) [3 pt] What is the output impedance of the following stage?



$R_{out} = R_{ON} \parallel R_{op}$

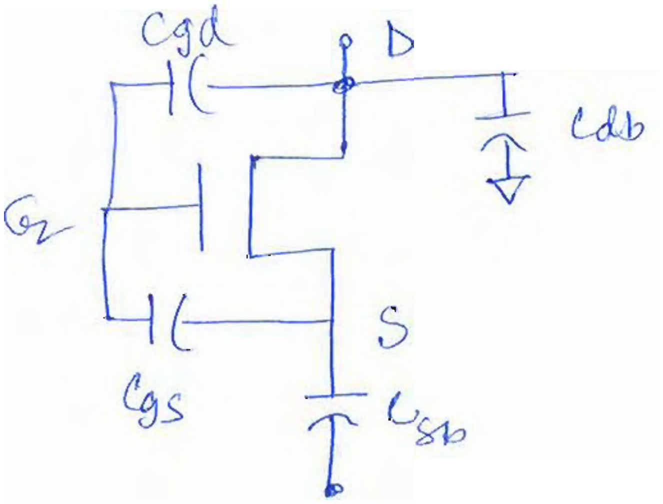
$R_{ON} = r_{o2} + (1 + g_{m_2} r_{o2})(r_{o1})$

$\approx g_{m_2} r_{o2} r_{o1}$

$R_{op} \approx g_{m_3} r_{o3} r_{o4}$

$\therefore \boxed{R_{out} \cong (g_{m_2} r_{o2} r_{o1}) \parallel (g_{m_3} r_{o3} r_{o4})}$

(b) [3 pt] Draw all the capacitances (relevant to the small signal response) of a MOSFET clearly identifying their physical origins.



C_{gd} = fringe cap

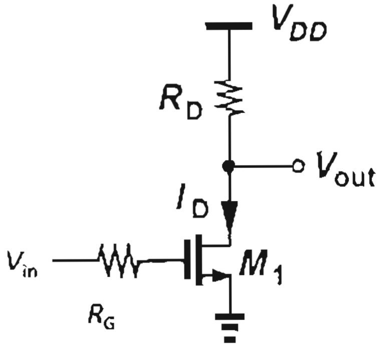
C_{gs} = fringe cap + 2/3 C_{ox}

C_{db} = Cap between drain and bulk

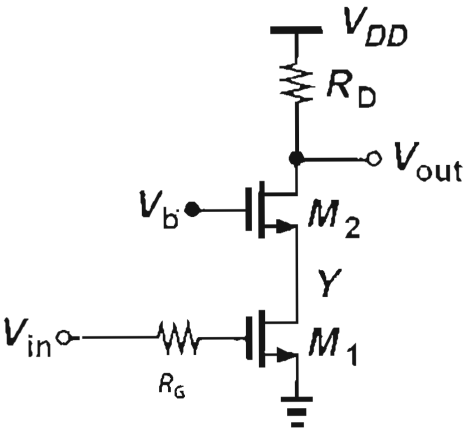
C_{sb} = cap between source and bulk.

(c) [5 pt]If M_1 and M_2 are identical transistors, which of the following will operate at a higher frequency than the other? Show by comparing each one's pole frequencies.

(a)



(b)



$$\omega_{in_a} \propto \frac{1}{C_{gs} + C_{gd}(1+|A_v|)}$$

$$|A_v| = g_m R_D$$

$$\omega_{in} \propto \frac{1}{C_{gs} + C_{gd}(1+|A_{v_2}|)}$$

$$|A_{v_2}| \cong g_{m_1}\left[\frac{1}{g_{m_2}}\right] \approx 1$$

$$\omega_{in_b} \propto \frac{1}{C_{gs} + 2C_{gd}}$$

Hence, $\boxed{\omega_{in_b} \gg \omega_{in_a}}$