**Problem 1** [15 points]: Semiconductor Basics

Consider a Si sample of length 10 μm and cross-sectional area 1 μm², uniformly doped with 10¹⁶ cm⁻³ boron, maintained at T = 300K. 1 Volt is applied across its length, as shown below:

![Diagram of silicon sample with 1 Volt applied](image)

a) Estimate the resistance of this sample. [6 pts]

\[ N_A = 10^{16} \text{ cm}^{-3}, \quad N_D = 0 \]

Since \( N_A > N_D \), this sample is p-type:

\[ p = N_A - N_D = 10^{16} \text{ cm}^{-3} \]

\[ n = \frac{n_i^2}{p} = \frac{10^{20}}{10^{16}} = 10^4 \text{ cm}^{-2} \]

From plot on Page 1, \( \mu_p \approx 450 \text{ cm}^2/\text{V.s} \) and \( \mu_n \approx 1200 \text{ cm}^2/\text{V.s} \)

\[ \rho \approx \frac{1}{\frac{1}{\mu_p} + \frac{1}{\mu_n}} = \frac{1}{(1.6 \times 10^{-19})(450)(10^4)} \approx 1.4 \Omega \cdot \text{cm} \]

\[ R = \rho \frac{L}{A} = 1.4 \frac{10 \times 10^{-4}}{(10^{-4})^2} = 1.4 \times 10^5 \Omega = 140 \Omega \]

b) Estimate the electron drift velocity. [5 pts]

\[ \mathcal{E} = \frac{1 \text{ V}}{10 \times 10^{-4} \text{ cm}} = 10^2 \text{ V/cm} \]

From plot on Page 1, \( \mu_n \approx 1200 \text{ cm}^2/\text{V.s} \)

\[ \mathbf{v}_e = \mu_n \mathcal{E} = 1200 \cdot 10^2 = 1.2 \times 10^6 \text{ cm/s} \]

c) Qualitatively (no calculations required), how would the resistivity of this sample change if it were to be additionally doped uniformly with 2x10¹⁶ cm⁻³ phosphorus? Explain briefly. [4 pts]

Since \( N_D > N_A \), this sample is now n-type, with \( n = N_D - N_A = 10^{16} \text{ cm}^{-3} \)

From the plot on Page 1, \( \mu_n \approx 900 \text{ cm}^2/\text{V.s} \), which is 2X greater than the hole mobility in the uncompensated sample.

Since the majority-carrier concentration is unchanged, and the majority-carrier mobility Page 3 is doubled, the resistivity is halved (i.e. \( \rho \) is reduced by a factor of 2.)
Problem 2 [15 points]: PN Junctions
Consider a Si PN junction diode, maintained at \( T = 300K \), with a structure and potential distribution as shown.

\[ \begin{align*}
V_0 &= V_T \ln \left( \frac{N_A N_D}{N_i^2} \right) = 0.026 \ln \left( \frac{10^{16} \times 10^{16}}{10^{20}} \right) = 0.026 \ln(10^{12}) \\
&= 12 \cdot 0.026 \ln(10) = 12 \cdot 0.06 = 0.72 \text{ V} \\
\end{align*} \]

a) Calculate the built-in potential, \( V_b \). [4 pts]

b) What is the applied voltage, \( V_D \)? Is this diode forward or reverse biased? (Circle one.) [7 pts]

\[ \begin{align*}
W_{dep} &= \sqrt{\frac{2 \varepsilon_{Si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_b - V_D)} = \sqrt{\frac{4 \varepsilon_{Si}}{\varepsilon N} (V_b - V_D)} \\
&= \sqrt{\frac{2 \varepsilon_{Si}}{q} W_{dep} N} = \frac{(1.6 \times 10^{-19})(0.4 \times 10^{-9})^2 (10^{16})}{4 \times 10^{-12}} = 0.64 \text{ V} \\
\end{align*} \]

\[ V_D = V_0 - V_D = 0.64 \text{ V} = 0.72 \text{ V} - 0.64 \text{ V} = 0.08 \text{ V} \]

Since \( V_D > 0 \), the diode is forward biased.

c) Calculate the areal junction (depletion) capacitance. [4 pts]

\[ C_{dep} = \frac{\varepsilon_{Si}}{W_{dep}} = \frac{10^{-12} \text{ F/cm}}{0.4 \times 10^{-4} \text{ cm}} = 2.5 \times 10^8 \text{ F/cm}^2 \]
Problem 3 [15 points]: Bipolar Junction Transistor Design

a) Why is the base region doped less heavily than the emitter region? [3 pts]

The ratio of carrier diffusion into the base (which determines $I_C$) to
carrier diffusion into the emitter (which determines $I_B$) is proportional
to $N_E/N_B$. Thus, to achieve large current gain $Q = I_C/I_B$, $N_E$ should be much larger than $N_B$.

b) Why is the base region doped more heavily than the collector region? [3 pts]

To minimize base-width modulation (i.e., to maximize the Early voltage $V_A$ and hence the BJT output resistance $r_o$), the width of the collector-
junction depletion region in the base should be minimized, by making $N_E$ much greater than $N_C$ so that most of the depletion region resides within the collector.

c) Indicate in the table below (by checking the appropriate box) how the BJT parameters would change, if the base width were to be increased (e.g., by 2x). Provide qualitative reasoning for your answers. [9 pts]

<table>
<thead>
<tr>
<th>BJT Parameter</th>
<th>Parameter will</th>
<th>Brief Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>increase</td>
<td>decrease</td>
</tr>
<tr>
<td>Reverse saturation current, $I_S$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Common-emitter DC current gain, $\beta$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Early voltage, $V_A$</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
Problem 4 [15 points]: Bipolar Junction Transistor I-V and Small-Signal Model
Consider a BJT with the I-V characteristics as shown below.

a) Draw the small-signal model for the BJT for the DC bias condition $V_{BE} = 0.8\text{V}$, $V_{CE} = 2.5\text{V}$. [12 pts]
(Indicate numerical values and units for $r_x$, $g_m$, and $r_o$, and label the transistor terminals.)

$g_m = \frac{I_c}{V_T} = \frac{3 \times 10^{-3} \text{A}}{0.026 \text{V}} \approx 0.1 \Omega$

$r_{\pi} = \frac{\beta}{g_m} \approx \frac{75}{0.1} = 750 \Omega$

$r_o = \text{the inverse slope of the } I_c \text{ vs. } V_{CE} \text{ characteristic} \approx \frac{5 \text{V}}{2 \text{mA}} = 2.5 \text{k}\Omega$

b) Show qualitatively (by sketching curves on each of the plots above) how the I-V characteristics would change if the emitter dopant concentration were to be increased by a factor of 2. [3 pts]

If $N_E$ increases by 2X, $I_B \propto \frac{1}{N_E}$ would decrease by 2X.

$I_c$ is not strongly dependent on $N_E$, so the $I_c$ vs. $V_{CE}$ characteristic would not be affected significantly.
**Problem 5 [20 points]: BJT Amplifier**

Consider the BJT amplifier stage shown below, operating at $T = 300$K with a bias current $I_C = 0.1$mA. Assume $I_S = 1 \times 10^{-16}$A, $\beta = 100$, $V_A = \infty$. $R_C = 10 \, k\Omega$, $R_E = 5 \, k\Omega$, and $V_{CC} = 2.5$V. Note that $e^{0.72/0.025} \approx 10^{12}$.

![BJT Amplifier Circuit Diagram]

a) What is the purpose of $R_1$ and $R_2$? [2 pts]

   to establish the **DC bias voltage for the base of the BJT**.

b) What is the purpose of $R_E$? [2 pts]

   to reduce the error in $I_C$ (hence $g_m$, $r_\pi$) resulting from errors in the values of $R_1$ and $R_2$.

c) Is the BJT operating in the active mode? Justify your answer. [5 pts]

   Since $I_C = 10^{12} I_S$, $V_{BE} = 0.72$V > 0  
   Since $I_E = I_C = 0.1$mA, the voltage dropped across $R_E$ is $(0.1mA)(5k\Omega) = 0.5$V 
   $\Rightarrow V_B = V_{RE} + V_{BE} = 0.5V + 0.72V = 1.22V$ 
   $V_C = V_{CC} - I_C R_C = 2.5 - (0.1mA)(10k\Omega) = 2.5V - 1V = 1.5V$ 
   Since $V_C > V_B$, collector junction is reverse-biased. ✓

d) Draw (in the box provided) the **most simplified circuit** that can be used for AC analysis to determine the small-signal voltage gain, $A_v$. You can assume that $C_1$ and $C_2$ are large, so that their impedances are negligible at the small-signal frequency of interest. Label the various circuit elements. [6 pts]

![Most Simplified Circuit Diagram]
Problem 5 (continued)
e) Write expressions for the small-signal voltage gain ($A_v$), input resistance ($R_{in}$), output resistance ($R_{out}$). [5 pts]

$$A_v = -g_m R_C$$  \hspace{1cm} \text{(from circuit in part (d))}

Circuit for analysis of $R_{in} = \frac{v_x}{i_x}$:

$$R_{in} = R_1 \parallel R_2 \parallel r_\pi$$

Circuit for analysis of $R_{out} = \frac{v_x}{i_x}$:

$$R_{out} = R_C$$