MIDTERM EXAMINATION #1
Time allotted: 80 minutes

NAME: ___________________________ Last ________________ First ________________ Signature ________________

STUDENT ID#: ___________________________

INSTRUCTIONS:
1. Use the values of physical constants provided below.
2. SHOW YOUR WORK. (Make your methods clear to the grader!)
3. Clearly mark (underline or box) your answers.
4. Specify the units on answers whenever appropriate.

PHYSICAL CONSTANTS

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic charge</td>
<td>$q$</td>
<td>$1.6 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k$</td>
<td>$8.62 \times 10^{-5}$ eV/K</td>
</tr>
<tr>
<td>Thermal voltage at 300K</td>
<td>$V_T = kT/q$</td>
<td>0.026 V</td>
</tr>
</tbody>
</table>

Note that $V_T \ln(10) = 0.060$ V at $T=300$K

PROPERTIES OF SILICON AT 300K

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band gap energy</td>
<td>$E_G$</td>
<td>1.12 eV</td>
</tr>
<tr>
<td>Intrinsic carrier concentration</td>
<td>$n_i$</td>
<td>$10^{10}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Dielectric permittivity</td>
<td>$\varepsilon_{si}$</td>
<td>$1.0 \times 10^{-12}$ F/cm</td>
</tr>
</tbody>
</table>

Electron and Hole Mobilities in Silicon at 300K

SCORE: 1 ______ / 25

2 ______ / 25

3 ______ / 30

Total: ______ / 80
Problem 1 [25 points]: Semiconductor Basics
a) Consider a Si sample of length 10 μm and cross-sectional area 1 μm², uniformly doped with 10^{18} cm⁻³ arsenic, maintained at T = 300K. 1 Volt is applied across its length, as shown below:

i) What are the electron and hole concentrations, \( n \) and \( p \), in this sample? [4 pts]

*Arsenic is a donor in silicon \( \Rightarrow N_D = 10^{18} \text{cm}^{-3} \), \( N_A = 0 \)*

Since \( N_D > N_A \), this sample is \( n \)-type.

\[
\begin{align*}
n &= N_D - N_A = 10^{18} \text{cm}^{-3} \\
p &= n^+ / n = 10^{20} \text{cm}^{-6} / 10^{18} \text{cm}^{-3} = 100 \text{cm}^{-2}
\end{align*}
\]

ii) Estimate the resistance of this sample. [5 pts] From plot on Page 1, \( \mu_n \approx 300 \text{cm}^2/\text{V} \cdot \text{s} \)

*resistivity \( \rho = \frac{1}{\mu_n n + \mu_p p} = \frac{1}{\mu_n n} \) since \( n \gg p \)*

\[
\rho = (1.6 \times 10^{-19} \text{C})(300 \text{cm}^2/\text{V} \cdot \text{s})(10^{18} \text{cm}^{-3}) = 0.02 \ \Omega \cdot \text{cm}
\]

resistance \( R = \frac{L}{A} = (0.02 \ \Omega \cdot \text{cm})(\frac{10 \times 10^{-4} \text{cm}}{1 \times 10^{-8} \text{cm}^2}) = 2000 \Omega \)

iii) Qualitatively (no calculations required), how would the resistance of this sample change if it were to be additionally doped with 2 × 10^{18} \text{cm}^{-3} boron? Explain briefly. [4 pts]

* Boron is an acceptor in silicon \( \Rightarrow N_A = 2 \times 10^{18} \text{cm}^{-3} \)*

The sample would be converted to \( p \)-type material with hole concentration \( p = N_A - N_D = 10^{18} \text{cm}^{-3} \) (same majority-carrier concentration as before).

Since the hole mobility is lower than the electron mobility, the resistance of the sample would increase.
b) Consider a Si pn junction diode, maintained at $T = 300K$, with a structure and $E$-field distribution as shown.

\[ V_D \]

\[ N_A = 10^{16} \text{cm}^{-3} \quad N_D = 10^{17} \text{cm}^{-3} \]

\[ E(x) \text{ [V/cm]} \]

\[ x \text{ [\mu m]} \]

\[ -3.2 \times 10^4 \]

i) Calculate the built-in potential, $V_0$. [4 pts]

\[
V_0 = V_T \ln \left( \frac{N_A}{n_i} \frac{N_D}{n_i} \right) = 0.026 \ln \left( \frac{10^{15}}{10^{20}} \frac{10^{17}}{10^{20}} \right) = 0.026 \ln (10^{12})
\]

\[
= 12 \times (0.026) \ln (10) = 12 \times 0.060 = 0.72 \text{V}
\]

ii) What is the applied voltage, $V_D$? Is this diode forward or reverse biased (circle one)? [4 pts]

The total potential dropped across the junction is $-\int E(dx)$, i.e. the area under the $E(x)$ distribution times $-1$.

From the plot of $E(x)$, this area is $\frac{1}{2} (2 \times 10^{-4} \text{cm}) (3.2 \times 10^4 \text{V/cm})$

\[ = -3.2 \text{V} \]

The total potential dropped across the junction is also $V_0 - V_D$:

\[ V_0 - V_D = 3.2 \text{V} \quad \Rightarrow \quad V_D = V_0 - 3.2 \text{V} = 0.72 \text{V} - 3.2 \text{V} = -2.48 \text{V} \]

iii) Calculate the areal junction capacitance. [4 pts]

\[
C_{dep} = \frac{\varepsilon_S i}{W_{dep}} = \frac{10^{-12} \text{F/cm}}{2 \times 10^{-4} \text{cm}} = 5 \times 10^{-9} \text{F/cm}^2
\]
Problem 2 [25 points]: Bipolar Junction Transistor

a) i) What is the Early effect (i.e. how is it manifested in the I-V characteristic of a BJT)? [2 pts]

The Early effect is an increase in $I_c$ with increasing $|V_{CE}|$, for a fixed $|V_{BE}|$.

ii) Why is the Early effect undesirable, for BJT amplifier applications? [2 pts]

The Early effect degrades small-signal voltage gain and decreases output resistance, each of which are undesirable for amplifier applications.

b) Indicate in the table below (by checking the appropriate box) how the BJT parameters would change, if the emitter doping were to be increased (e.g. by 2x). Provide qualitative reasoning for your answers. [9 pts]

<table>
<thead>
<tr>
<th>BJT Parameter</th>
<th>Parameter will</th>
<th>Brief Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>increase</td>
<td>decrease</td>
</tr>
<tr>
<td>Reverse saturation current, $I_s$</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Common-emitter DC current gain, $\beta$</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Early Voltage, $V_A$</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>
c) i) Accurately sketch on the plots below the $I_B$-$V_{CE}$ and $I_C$-$V_{CE}$ characteristics (for $0V < V_{CE} < 5V$) of an NPN BJT operating at $T = 300K$ with $I_s = 5 \times 10^{-15} \, \text{A}$, $\beta = 200$, and $V_A = 5V$, biased at $V_{BE} = 0.72V$. [6 pts]

Note that $e^{0.72/0.026} \approx 10^{12}$.

\[
I_C = I_S \frac{e^{V_{BE}/V_T}}{(1 + V_{CE}/V_A)} = (5 \times 10^{-15} \, \text{A}) \frac{e^{0.72/0.026}}{(1 + V_{CE}/5\,\text{V})} = (5 \times 10^{-15})(10^{12})(1 + V_{CE}/5\,\text{V}) = 5 \times 10^{-3} \, \text{A} \left(1 + \frac{V_{CE}}{5\,\text{V}}\right)
\]

For $V_{CE} = 0\,\text{V}$, $I_C = 5 \, \text{mA}$

\[I_B = I_C / \beta = \frac{5 \, \text{mA}}{200} = 25 \, \mu\text{A}\]

For $V_{CE} = 5\,\text{V}$, $I_C = 10 \, \text{mA}$

ii) Draw the small-signal model for this BJT, biased at $V_{BE} = 0.72V$ and $V_{CE} = 2.5V$. [6 pts]

Indicate numerical values for the small-signal parameters, and label the transistor terminals.

(Note: $r_o = V_A/I_{C,\text{nominal}}$ where $I_{C,\text{nominal}}$ is the collector current for $V_{CE} << V_A$)

At $V_{CE} = 2.5\,\text{V}$, $I_C = 5 \, \text{mA} \left(1 + \frac{2.5\,\text{V}}{5\,\text{V}}\right) = 7.5 \, \text{mA}$

\[g_m = \frac{I_C}{V_T} = \frac{7.5 \times 10^{-3} \, \text{A}}{0.026 \, \text{V}} \approx 0.29 \, \text{S}\]

\[r_T = \frac{\beta}{g_m} = \frac{200}{0.29 \, \text{S}} \approx 690 \, \Omega\]

\[r_o = \frac{V_A}{I_{C,\text{nominal}}} = \frac{5\,\text{V}}{5\,\text{mA}} = 1000 \, \Omega\]
Problem 3 [30 points]: BJT Amplifiers
a) Consider the BJT amplifier stage shown below, operating at $T = 300$K with a bias current $I_c = 0.1 \text{mA}$.
Assume $I_s = 1 \times 10^{-16} \text{A}$, $\beta = 100$, $V_A = 50\mu \text{V}$.

If $I_B \ll I_I$,
then $V_b = \frac{5}{10+5} (3 \text{V}) = 1 \text{V}$

Check assumption:
$I_i = \frac{3 \text{V} - 1 \text{V}}{10 \text{K} \Omega} = 200 \mu \text{A} \gg I_B$

i) What is the value of $R_E$? [6 pts]
$I_c = I_S e^{V_BE/V_T} \quad \Rightarrow \quad V_{BE} = V_T \ln \left( \frac{I_c}{I_S} \right) = 0.026 \ln \left( \frac{10^{-4}}{10^{-16}} \right) = 0.026 \ln (10^{12}) = 12 \times 0.026 \ln (10) = 0.72 \text{V}$

$I_E = I_c + \frac{I_c}{\beta} \approx I_c = 0.1 \text{mA}$

$V_b = V_{BE} + I_E R_E$ \quad $\Rightarrow$ \quad $R_E = \frac{V_b - V_{BE}}{I_E} = \frac{1 \text{V} - 0.72 \text{V}}{0.1 \times 10^{-3} \text{A}} = 2.8 \text{K} \Omega$

ii) For what range of $R_C$ values is the BJT operating in the active mode? [4 pts]

For the BJT to be in active mode, $V_{out} \geq V_b$:

$V_{cc} - I_c R_C \geq V_b \quad \Rightarrow \quad R_C \leq \frac{V_{cc} - V_b}{I_c} = \frac{3 \text{V} - 1 \text{V}}{10^{-4} \text{A}} = 20 \text{K} \Omega$

iii) Draw (in the box provided) the most simplified circuit that can be used for AC analysis to determine $A_v$ for $R_C = 10 \text{K} \Omega$. $C_i$ is large, so that its impedance is negligible at the small-signal frequency of interest.

Indicate numerical values for the various circuit elements, but DO NOT SOLVE FOR $A_v$. [6 pts]
b) Consider the circuit below:

i) Is this a common emitter, common base, or emitter follower circuit? Justify your answer. [2 pts]

- Input signal is applied to the emitter.  
- Output signal is taken from the collector.  

\[ \Rightarrow \text{common base topology} \]

ii) Derive expressions for the voltage gain \((A_v)\), input resistance \((R_{in})\), and output resistance \((R_{out})\). [8 pts]

You may assume that the capacitors \(C_1\) and \(C_B\) are large, so that their impedances are negligible at the small-signal frequency of interest. You may also neglect the Early effect (i.e. assume \(V_A = \infty\)).

With the capacitors shorted, the circuit becomes:

\[
A_v = \frac{R_c}{R_s + R_c} \cdot \frac{R_E}{R_s + R_E} \\
R_{in} = \frac{1}{g_m} \parallel R_E \\
R_{out} = R_c
\]

iii) Describe one of the design trade-offs involved, when selecting the value of \(R_c\). [3 pts]

To achieve high voltage gain, \(R_c\) should be large - but then a large \(R_c\) results in larger \(R_{out}\) (which is undesirable for an amplifier) and reduced headroom (limiting the output voltage swing).