UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

EE 42 / 100 — Midterm 1 — Spring 2008

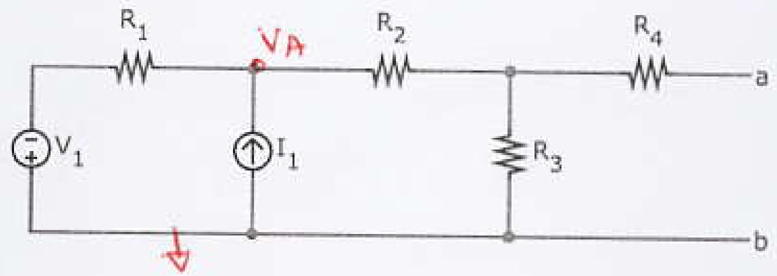
Name: Solutions	
SID:	
Section:	
Name of student left of you:	
Name of student right of you:	

Problem	Score
1	
2	
3	
4	
Total	

- Check the units of your results.
- Closed book, closed notes.
- No calculators.
- Leave pack, books, and electronic devices (e.g. cell phones) in isle.
- Take off caps or hats.
- Copy your answers into marked boxes on exam sheets.
- Simplify numerical and algebraic results as much as possible. Up to 10 points penalty for results that are not reasonably simplified.
- Be kind to the graders and write legibly. No credit for illegible results.
- No credit for multiple differing answers to the same question.
- The UC rules on dishonesty apply.

1. [25 points] Calculate the component values of Thévenin and Norton equivalents for the circuit shown below.

Note, V_1 is "flipped" →



$V_{\text{Thévenin}} =$	$(I_1R_1 - V_1)\cdot\dfrac{R_3}{R_1+R_2+R_3}$
$R_{\text{Thévenin}} =$	$\dfrac{(R_1+R_2)\cdot R_3}{R_1+R_2+R_3} + R_4 = \dfrac{(R_1+R_2)R_3 + R_4(R_1+R_2+R_3)}{(R_1+R_2+R_3)}$
$I_{\text{Norton}} =$	$\dfrac{(I_1 - R_1V_1)(R_3)}{(R_1+R_2)R_3 + R_4(R_1+R_2+R_3)}$
$R_{\text{Norton}} =$	$R_{th} = \dfrac{(R_1+R_2)R_3}{R_1+R_2+R_3} + R_4$

① Find open-circuit voltage
- use NVA
- Note no current flows through R_4

Node Voltage Analysis at node A

$$\frac{V_A + V_1}{R_1} - I_1 + \frac{V_A}{R_2+R_3} = 0$$

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2+R_3}\right) = I_1 - \frac{V_1}{R_1}$$

$$V_A = \left(I_1 - \frac{V_1}{R_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2+R_3}\right)^{-1}$$

$$V_A = (I_1R_1 - V_1)\left(\frac{R_2+R_3}{R_1+R_2+R_3}\right)$$

Voltage Divider

$$V_{ab} = V_A \cdot \frac{R_3}{R_2+R_3}$$

$$V_{ab} = (I_1R_1 - V_1)\cdot\frac{R_3}{R_1+R_2+R_3}$$

Prob 1) continued

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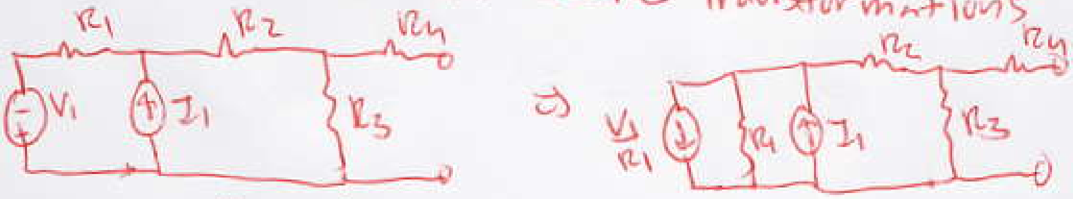
② Find R_{th}

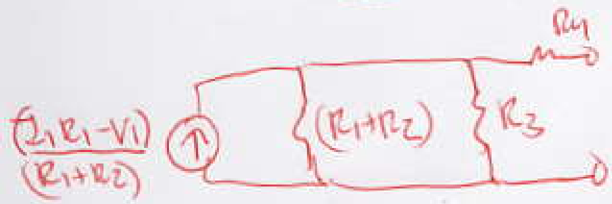
Deactivate sources (open-circuit current, short circuit voltage)

$$R_4 + (R_1+R_2) \| R_3 = R_4 + \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$$

③ Find $I_n = \dfrac{V_{th}}{R_{th}}$

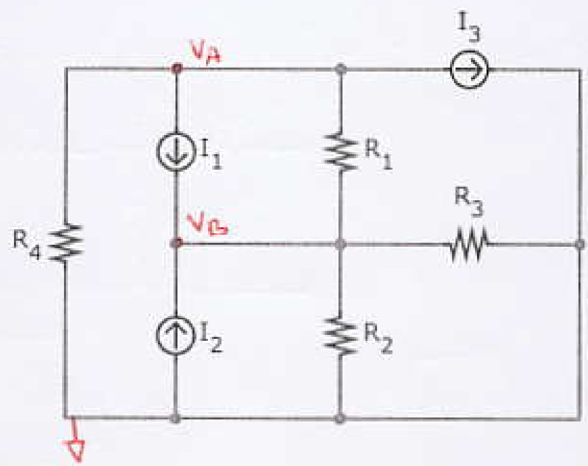
④ Alternative method: Source transformations





$$\Rightarrow \frac{(I_1R_1-V_1)}{(R_1+R_2)} \cdot \frac{(R_1+R_2)R_3}{R_3+R_1+R_2} = V_{th}$$

2. [25 points] Derive an expression for the power dissipated in source I_1.



$P_1 =$ See boxed answers

Setup
- $P_1 = I_1 (V_A - V_B)$ [Power dissipated by I_1]
- NVA to find V_A & V_B

@ V_A:

$$\frac{V_A}{R_4} + I_1 + \frac{(V_A - V_B)}{R_1} + I_3 = 0 \quad ①$$

@ V_B:

$$-I_2 + (-I_1) + \frac{V_B}{R_2 \| R_3} + \frac{V_B - V_A}{R_1} = 0 \quad ②$$

- At this point, you have to solve a system of linear equations. There are many ways to do this but the solution may get messy if you try a simple substitution method, with the given coefficients.

→

Problem 2

First, let's rearrange eqns. ① + ② so that the coefficients are clear

③ V_A from ①

$$V_A \underbrace{(R_1+R_4)}_{A} + V_B \underbrace{(-R_4)}_{B} + \underbrace{R_4 R_1 (I_1+I_3)}_{C} = 0$$

④ V_B from ②

$$V_A \underbrace{(-R_2 \| R_3)}_{X} + V_B \underbrace{(R_1 + R_2 \| R_3)}_{Y} \underbrace{-(I_2+I_1)(R_2 \| R_3) \cdot R_1}_{Z} = 0$$

Now, let's remove some of the messiness of this system by solving for a system with generic coefficients as defined above.

$$\begin{cases} V_A \cdot A + V_B \cdot B + C = 0 & \text{from ①} \\ V_A \cdot X + V_A \cdot Y + Z = 0 & \text{from ②} \end{cases}$$

$A = R_1 + R_4$ $\qquad$ $X = -(R_2 \| R_3)$

$B = -R_4$ $\qquad$ $Y = R_1 + (R_2 \| R_3)$

$C = R_4 R_1 (I_1 + I_3)$ $\qquad$ $Z = -(I_2 + I_1)(R_2 \| R_3) \cdot R_1$

③ $V_A = -\dfrac{B \cdot V_B - C}{A}$ $\qquad$ ④ $V_B = \dfrac{-A \cdot V_A - C}{B}$

& a sub ③ + ④ into ②

$$V_A = \frac{Z \cdot B - Y \cdot C}{X \cdot B - Y \cdot A} \qquad V_B = \frac{Z \cdot A - X \cdot C}{Y \cdot A - X \cdot B}$$

Now you can "simply" substitute in the coefficients and you have solved for V_A + V_B.

$$V_A = \frac{Z \cdot B - Y \cdot C}{X \cdot B - Y \cdot A}$$

$$= \frac{-(I_2 + I_1)(R_2 \| R_3)(-R_4)R_1 - [R_1 + (R_2 \| R_3)] R_4 R_1 (I_1 + I_3)}{-(R_2 \| R_3)(-R_4) - [R_1 + (R_2 \| R_3)](R_1 + R_4)}$$

Using $R_2 \| R_3 = \dfrac{R_2 \cdot R_3}{R_2 + R_3}$

$$\boxed{V_A = \frac{+(I_2 + I_1)(R_2 R_3) R_4 R_1 - (R_1(R_2 + R_3) + R_2 R_3) R_4 R_1 (I_1 + I_3)}{+ R_2 R_3 \cdot R_4 - [R_1 \cdot (R_2 + R_3) + R_2 \cdot R_3](R_1 + R_4)}}$$

units $= \dfrac{IR^4}{R^4} = IR$ ✓
(V)

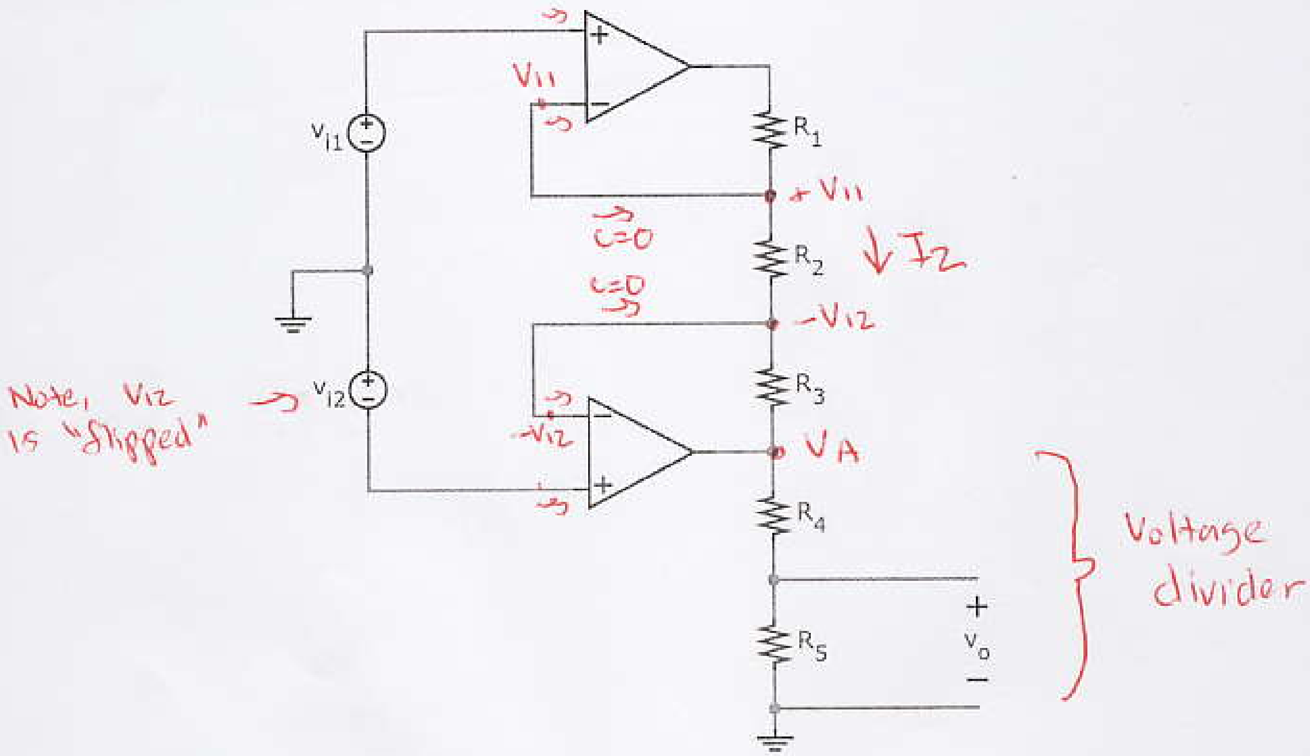
$$V_B = \frac{Z \cdot A - X \cdot C}{Y \cdot A - X \cdot B}$$

$$= \frac{-(I_2 + I_1)(R_2 \| R_3) \cdot R_1 \cdot (R_1 + R_4) - (-R_2 \| R_3) R_4 R_1 (I_1 + I_3)}{[R_1 + (R_2 \| R_3)](R_1 + R_4) - (-R_2 \| R_3)(-R_4)}$$

$$\boxed{V_B = \frac{-(I_2 + I_1) R_2 \cdot R_3 R_1 (R_1 + R_4) + (I_1 + I_3) R_2 R_3 R_4 R_1}{[R_1(R_2 + R_3) + R_2 \cdot R_3](R_1 + R_4) - R_2 R_3 R_4}}$$

$$\boxed{P = I_1 (V_A - V_B)}$$

3. [25 points] Derive an expression for v_o as a function of v_{i1} and v_{i2} and circuit parameters. Assume that the operational amplifier is ideal.
Suggestion: do not use node voltage analysis.



$$v_o = -\frac{R_5}{R_4+R_5}\left[V_{i2} + (V_{i1}+V_{i2})\cdot\frac{R_3}{R_2}\right]$$

$V_{+1} = V_{-1} = V_{i1}$

$V_{+2} = V_{-2} = -V_{i2}$

$$I_2 = \frac{V_{i1}-(-V_{i2})}{R_2} = \frac{V_{i1}+V_{i2}}{R_2}$$

$$V_A = -V_{i2} - I_2 R_3$$

(i) $$= -V_{i2} - \left(\frac{V_{i2}+V_{i1}}{R_2}\right)R_3$$

(ii) $$V_o = \frac{R_5}{R_4+R_5}\cdot V_A = \frac{R_5}{R_4+R_5}\left[-V_{i2} - \frac{(V_{i2}+V_{i1})}{R_2}\cdot R_3\right]$$

Voltage divider

$$= \frac{-R_5}{R_4+R_5}\left[V_{i2} + (V_{i2}+V_{i1})\cdot\frac{R_3}{R_2}\right]$$

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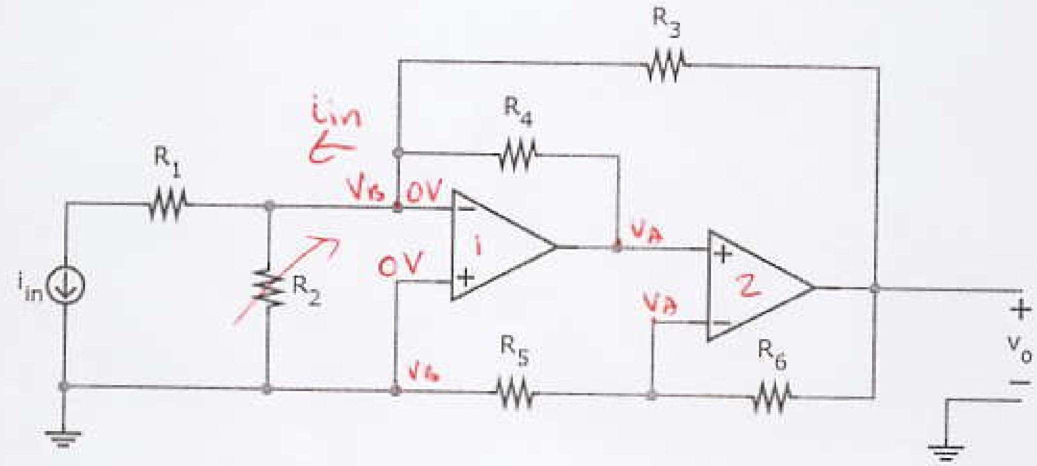
4. [25 points] Derive an expression for $\frac{v_o}{i_{in}}$ as a function of circuit parameters. Assume that the operational amplifiers are ideal.



$$\frac{v_o}{i_{in}} = \frac{(R_5+R_6)\,R_3\cdot R_4}{R_4\cdot R_5 + R_4\cdot R_6 + R_5\cdot R_3}$$

$V_B = 0\,V$

$V_A = V_o \cdot \frac{R_5}{R_5+R_6}$ (Voltage Divider)

- no current through R_2
- R_1 irrelevent since in series w/ current source

NVA @ node B

$$I_{in} + \frac{0-V_A}{R_4} + \frac{0-V_o}{R_3} = 0$$

$$R_3\cdot R_4\cdot I_{in} - V_A\cdot R_3 - V_o\cdot R_4 = 0$$

$$R_3\cdot R_4\cdot I_{in} - V_o\left[\frac{R_5}{R_5+R_6}\right]\cdot R_3 - V_o\cdot R_4 = 0$$

$$V_o = \frac{R_3\cdot R_4\cdot I_{in}}{\left[R_4 + \frac{R_5\cdot R_3}{R_5+R_6}\right]}$$

$$\frac{V_o}{I_{in}} = \frac{(R_5+R_6)\cdot R_3\cdot R_4}{R_4\cdot R_5 + R_4\cdot R_6 + R_5\cdot R_3}$$